

**MAT 2377**  
**Winter 2020**  
**Final Exam**  
**April 20, 2020**  
**Time Limit: 3 Hours**

**Final Examination**  
**Student: FALTES M**  
**Version: 91050**

**Start Time: 2PM EST**

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This exam contains 20 pages (including this cover page and tables) and 32 questions.

**Read carefully:** if you submit answers for this online examination, you promise and certify that

- all work done during the assigned exam period will be done entirely by yourself, with no help from others;
- you will not communicate with anybody except the professor during the exam, for exam- and course-related questions;
- you will not consult any people, sources, or writings other than the course textbook, your self-made review sheets, your course notes, your prepared answers, the course documents made available on Brightspace, and the exam itself;
- you will not provide information related to the exam's contents to other people until 24 hours after the exam is over.

Obviously, we do not have a way to monitor your adherence to these instructions (nor do we really wish to look for one); we will be asking, instead, for your word of honour that you will follow the directives and work on this examination alone.

Multiple choice questions and true or false question are worth 1 mark each; short answer questions, 2 marks. There are no part marks. For each question, record your answer in the corresponding space on the answer sheet that has been provided separately (save your file regularly). For multiple choice questions, select the closest answer among the options, when appropriate; for short answer questions, provide an answer to at least 2 decimals.

**IMPORTANT:** every student has been assigned a different exam (based on the last 5 digits of your student number). You need to answer the questions of the final exam that has been specifically assigned to you.

When you have answered all the questions and you are ready to upload the answers to Brightspace, save your answers to the file **Answer\_Sheet\_91050.pdf**. We suggest that you print and scan or take a screenshot (with time stamp) of your completed answer sheet before uploading to Brightspace (see the appropriate location in the “Assignment” section where you uploaded your 3rd assignment), and that you keep a copy of these documents (with time stamp). Should Brightspace's upload functionality not work when you are ready to submit, wait a little bit and try again; as a last recourse, please email your answer sheet with saved answers to [pboily@uottawa.ca](mailto:pboily@uottawa.ca) if you are registered in section A or [iyadegar@uottawa.ca](mailto:iyadegar@uottawa.ca) if you are registered in section B.

No worries. You've got this.

**1. Multiple Choice Question**

In a nickel-cadmium battery NiCd, a fully charged cell consists of nickelic hydroxide. Nickel is an element which has multiple oxidation states. Let  $X$  be the charge of nickel, which has the following probability mass function:

| $x$ | $f_X(x)$ |
|-----|----------|
| 0   | 0.18     |
| 1   | 0.34     |
| 2   | 0.33     |
| 4   | 0.15     |

Determine the mean charge of nickel.

A. 2.0   B. 1.5   C. 1.6   D. 1.45   E. none of the preceding

**2. Multiple Choice Question**

Previous experience has shown that the fracture resistance of a wire used in the manufacture of drapery is normally distributed with  $\sigma^2 = 2$ . A random sample of 25 specimens have been examined and they yielded a mean resistance of  $\bar{x} = 98$  psi. Give a 95% confidence interval for the mean resistance.

A. [97.216,98.784]   B. [97.216,98.554]   C. [97.456,98.784]   D. [97.446,98.554]  
E. none of the preceding

**3. Multiple Choice Question**

Here is a random sample for the ignition time (in seconds) for 4 upholstery materials exposed to heat: 2.58, 2.52, 4.04, 2.20. Obtain a point estimate for the population standard deviation  $\sigma$  of the ignition time.

A. 0.125   B. 2.835   C. 0.8205   D. 0.6732   E. none of the preceding

**4. Multiple Choice Question**

Trucks arrive at a loading/unloading station according to a Poisson process with a rate of 2 trucks per hour. A truck just arrived at the station. A station employee would like to know if he has enough time to take a break. Let the random variable  $T$  represent the waiting time (in hours) until the next truck arrival. What is the probability that the employee has 15 minutes until the next arrival (that is, determine  $P(T > 0.25)$ )?

A. 0.5   B. 0.3435   C. 0.6065   D. 0.7788   E. none of the above

**5. Multiple Choice Question**

Assume that the yearly rainfall in Winnipeg is normally distributed with mean 35.4 inches and standard deviation 4.2 inches. A university student is in Winnipeg for 4 years. Assume that yearly rainfall are independent. Let  $D = X_1 - X_4$  be the difference of the rainfall in year 1 and in year 4. Compute the mean and the standard deviation of  $D$ , respectively.

- A. 0; 8.4   B. 0; 5.9   C. 0; 0   D. -35.4; 5.9   E. -35.4; 8.4

**6. Multiple Choice Question**

In a pile of 20 tires, three of them are defective. Four tires are selected randomly for inspection. What is the probability that exactly one is defective?

- A. 0.15   B. 0.42   C. 0.1053   D. 0.4912   E. none of the preceding

**7. Multiple Choice Question**

Consider the following sample: 16, 13, 10, 17, 16, 14, 5, 12, 15, 8, 9, 10, 15, 22, 8. Calculate the variance and interquartile range of the sample, respectively.

- A. 19.38; 7   B. 15.7; 7   C. 4.40; 7   D. 19.38; 7.5   E. 19.38; 6.5

**8. Multiple Choice Question**

Previous experience has shown that the breaking strength of the fabric used in a certain brand of drapes is normally distributed with a mean of 97 pounds per square inch. A random sample of 9 specimens is examined to reveal an average breaking strength of  $\bar{x} = 98$  pounds per square inch and a standard deviation of 2 pounds per square inch are observed. The  $p$ -value to test the hypothesis that the true mean is not 97 is between

- A. (0.05,0.1)   B. (0.1,0.2)   C. (0.025,0.05)   D. [0,0.025)  
E. none of the preceding

**9. Multiple Choice Question**

A new treatment has been developed for a certain type of cement, resulting in a mean pressure resistance of 5000kg/cm<sup>2</sup> and a standard deviation of 130kg/cm<sup>2</sup>. To verify the null hypothesis of  $\mu = 5000$  against the alternative of  $\mu < 5000$ , a random sample of 50 pieces of cement are examined. Determine the probability of committing a Type II error if  $\mu = 4960$  and  $\alpha = 0.05$ .

- A. 0.298   B. 0.954   C. 0.082   D. 0.105   E. none of the preceding

## 10. Multiple Choice Question

Consider the discrete random variables  $X$  and  $Y$  with the following joint probability mass function:

| $x$ | $y$ | $f_{XY}(x, y)$ |
|-----|-----|----------------|
| -1  | 0   | 1/8            |
| 0   | -1  | 1/4            |
| 0   | 1   | 1/4            |
| 1   | 0   | 1/8            |
| -1  | 1   | 1/8            |
| 1   | -1  | 1/8            |

Compute the covariance  $\text{Cov}(X, Y) = E[XY] - E[X]E[Y]$ .

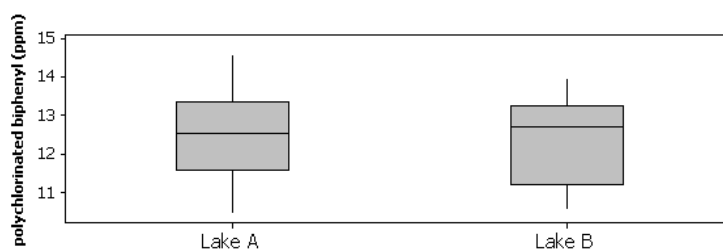
- A. 7/8   B. 3/8   C. 1/4   D. -1/4   E. none of the preceding

## 11. Multiple Choice Question

A sample of 20 fish are captured in lake  $A$  and their concentrations of polychlorinated biphenyl are measured using a certain technique. Furthermore, 15 fish are captured in lake  $B$  and their concentrations of polychlorinated biphenyl are measured. The data (in parts per million) are summarized as follows:

|                    | Lake $A$ | Lake $B$ |
|--------------------|----------|----------|
| mean               | 12.42    | 12.50    |
| standard deviation | 1.167    | 1.141    |
| sample size        | 20       | 15       |

Boxplots are constructed for each lake:



Is it reasonable to assume that

- A. the populations are normal with equal variances  
 B. the populations are normal with unequal variances  
 C. only one of the populations is normal  
 D. the populations are not normal  
 E. none of the preceding

**12. Multiple Choice Question**

It was reported that 60% of a particular type of services are financed by private funds, 70% are funded by employers and employees, and 40% are financed by private funds and by the employers and employees. What is the probability that an individual that is randomly accessing such a service will choose a service that is funded by employers and employees but not by private funds?

- A. 0.30   B. 0.20   C. 0.10   D. 0.50   E. none of the preceding

**13. Multiple Choice Question**

A machine produces cylindrical metal pieces. A random sample of the pieces yields diameters

1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.04, 1.03, 1.01.

Determine a 99% confidence interval for the mean diameter. You may assume the diameters are normally distributed.

- A. [0.989, 1.022]   B. [0.983, 1.035]   C. [0.991, 1.034]   D. [0.987, 0.024]  
E. none of the above

**14. Multiple Choice Question**

It is known that a particular company produces products of which 30% are defective. We select items at random and identify it as being defective or not. Calculate the probability that the sixth selected item will be the 3rd defective.

- A. 0.38282   B. 0.09261   C. 0.24518   D. 0.75494   E. none of the above

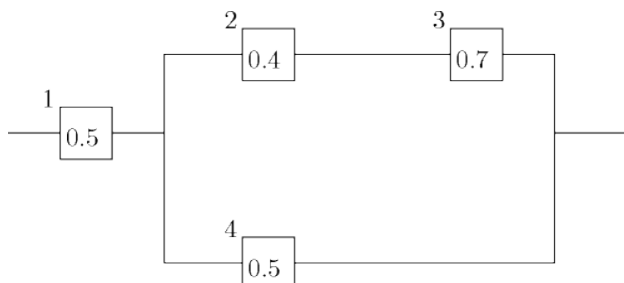
**15. Multiple Choice Question**

Let  $X$  equal the excess weight of a "1000 grams" bottle of soap. Assume that  $X$  follows a normal distribution with variance  $169\text{g}^2$ . What sample size is required to have a level of confidence of 95% that the maximum error of the estimate of the mean of the excess weight is less than 1.5g?

- A. 302   B. 287   C. 289   D. 301   E. 288

**16. Multiple Choice Question**

Consider the following system with four components. It operates only if there is a path of functional components from left to the right. The probability that each component functions is as shown. What is the probability that the circuit operates? Assume independence.



- A. 0.32   B. 0.16   C. 0.035   D. 0.68   E. none of the preceding

**17. Multiple Choice Question**

The lifetime (in 1,000 km) for a certain brand of tire is modeled as a continuous random variable with the following cumulative distribution function:  $F(x) = 1 - e^{-(x-10)/50}$ ,  $x > 10$ . Determine a lifetime  $x$  (in 1,000 km) that will be exceeded by 60% of the tires.

- A. 44.66   B. 50   C. 55.81   D. 35.54   E. none of the above

**18. Multiple Choice Question**

The lifetime of a voltage regulator for a car follows an exponential distribution with a mean of 6 years. The number of failures follows a Poisson distribution with a mean of one failure every 6 years. A person buys a 6-year-old car and counts on owning it for another 6 years. If the regulator suffers a failure 3 years after the purchase, compute the average number of years until the next failure.

- A. 6   B. 0   C. 3   D. 12   E. none of the preceding

**19. Multiple Choice Question**

A random sample of size  $n_1 = 16$  is selected from a normal population with mean 75 and variance 288. A second random sample of size  $n_2 = 9$  is selected from a normal population with mean 80 and variance 162. Assume that the random samples are independent. Let  $\bar{X}_1$  and  $\bar{X}_2$  be the respective sample means. Find the probability that  $\bar{X}_1 + \bar{X}_2$  is larger than 156.5.

- A. 0.5987   B. 0.4013   C. 0.6231   D. 0.4235   E. none of the preceding

**20. Multiple Choice Question**

Here are the waiting times (in minutes) for a bus for a particular person on 5 consecutive working days: 10, 1, 13, 9, 5 (presumably this was not happening in Ottawa...) Compute the mean and the variance of the sample, respectively.

- A. 7.6; 15.5    B. 7.6, 21.8    C. 6.9, 21.8    D. 6.9, 15.5  
E. none of the preceding

**21. True or False Question**

The length of time in minutes between consecutive calls to 911 in a small city has density

$$f(x) = \begin{cases} \frac{1}{20} e^{-x/20} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

The probability that the time between consecutive calls is greater than 20 minutes is thus  $1/e$ .

True    False

**22. True or False Question**

A new treatment has been developed for a certain type of cement. To test the null hypothesis that  $\mu = 5000$  against the alternative that  $\mu \neq 5000$ , where  $\mu$  is the mean resistance in  $\text{kg}/\text{cm}^2$ , a sample of 50 samples of cement is examined and we observe  $\bar{x} = 4775$  and  $s = 130$ . At  $\alpha = 0.05$ , the evidence against the null hypothesis in favour of the alternative hypothesis is significant.

True    False

**23. True or False Question**

In order to establish a control chart for the mean of a process, 20 samples each of size 4 are collected. We note that  $\sum_{i=1}^{20} \bar{x}_i = 4000$  and  $\sum_{i=1}^{20} s_i = 500$ . The value of the upper control limit of the chart for the mean is approximately equal to 217.

True    False

**24. True or False Question**

The percentage of males in 1986 who are 18-19 years old and married was 3.7%. To test whether or not this percentage had increased in 2015, a random sample of 300 males aged 18-19 was taken; 20 of which were married. Using a level of significance of  $\alpha = 0.05$ , we test the hypothesis  $H_0$  that the proportion that is married is 3.7% against the alternative that it is larger. Based on the data, the observed value of the corresponding test statistic is 2.722 and we reject  $H_0$ .

True    False

**25. Short Answer Question**

We have a dataset with  $n = 10$  pairs of observations  $(x_i, y_i)$ , and

$$\begin{aligned} \sum_{i=1}^n x_i &= 683, \quad \sum_{i=1}^n y_i = 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731. \end{aligned}$$

What is an approximate 99% confidence interval for the mean response at  $x_0 = 90$ ?

**26. Short Answer Question**

We have a dataset with  $n = 10$  pairs of observations  $(x_i, y_i)$ , and

$$\begin{aligned} \sum_{i=1}^n x_i &= 683, \quad \sum_{i=1}^n y_i = 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731. \end{aligned}$$

What is the coefficient of correlation for this data?

**27. Short Answer Question**

We have  $m = 5$  preliminary samples of size  $n = 3$  (some numbers have unfortunately been erased by accident by a clumsy co-op student):

| $i$    | $x_{i,1}$ | $x_{i,2}$ | $x_{i,3}$ | $\bar{x}_i$ | $r_i$ | $s_i$ |
|--------|-----------|-----------|-----------|-------------|-------|-------|
| 1      | 27.1      | 29.4      |           | 27.9        |       | 1.3   |
| 2      | 30.6      | 32.5      | 32.4      | 31.83       | 1.9   | 1.07  |
| 3      | 25.7      | 35.5      | 30        | 30.4        |       | 4.91  |
| 4      | 31.1      | 23.2      | 25        | 26.43       | 7.9   |       |
| 5      | 24.1      | 34.2      | 27.4      | 28.57       | 10.1  | 5.15  |
| total: |           |           |           | 145.13      | 32    | 16.57 |

What is the control chart (give the interval) for  $\bar{X}$  from  $\bar{S}$ ?



**28. Short Answer Question**

We have a dataset with  $n = 10$  pairs of observations  $(x_i, y_i)$ , and

$$\sum_{i=1}^n x_i = 683, \quad \sum_{i=1}^n y_i = 813,$$

$$\sum_{i=1}^n x_i^2 = 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731.$$

What is an approximate 99% prediction interval for the response  $y_0$  at  $x_0 = 90$ ?

**29. Short Answer Question**

We have  $m = 5$  preliminary samples of size  $n = 3$  (some numbers have unfortunately been erased by accident by a clumsy co-op student):

| $i$    | $x_{i,1}$ | $x_{i,2}$ | $x_{i,3}$ | $\bar{x}_i$ | $r_i$ | $s_i$ |
|--------|-----------|-----------|-----------|-------------|-------|-------|
| 1      | 27.1      | 29.4      |           | 27.9        |       | 1.3   |
| 2      | 30.6      | 32.5      | 32.4      | 31.83       | 1.9   | 1.07  |
| 3      | 25.7      | 35.5      | 30        | 30.4        |       | 4.91  |
| 4      | 31.1      | 23.2      | 25        | 26.43       | 7.9   |       |
| 5      | 24.1      | 34.2      | 27.4      | 28.57       | 10.1  | 5.15  |
| total: |           |           |           | 145.13      | 32    | 16.57 |

What is the control chart (give the interval) for  $\bar{X}$  from  $\bar{R}$ ?

**30. Short Answer Question**

We have a dataset with  $n = 10$  pairs of observations  $(x_i, y_i)$ , and

$$\sum_{i=1}^n x_i = 683, \quad \sum_{i=1}^n y_i = 813,$$

$$\sum_{i=1}^n x_i^2 = 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731.$$

What is an approximate 95% confidence interval for the slope of the line of best fit?

**31. Short Answer Question**

We have a dataset with  $n = 10$  pairs of observations  $(x_i, y_i)$ , and

$$\sum_{i=1}^n x_i = 683, \quad \sum_{i=1}^n y_i = 813,$$

$$\sum_{i=1}^n x_i^2 = 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731.$$

What is the line of best fit for this data?

**32. Short Answer Question**

We have a dataset with  $n = 10$  pairs of observations  $(x_i, y_i)$ , and

$$\begin{aligned} \sum_{i=1}^n x_i &= 683, \quad \sum_{i=1}^n y_i = 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731. \end{aligned}$$

What is an approximate 95% confidence interval for the intercept of the line of best fit?



**Table A.1** (continued) Binomial Probability Sums  $\sum_{x=0}^r b(x; n, p)$ 

| <i>n</i> | <i>r</i> | <i>p</i> |        |        |        |        |        |        |        |        |        |
|----------|----------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|          |          | 0.10     | 0.20   | 0.25   | 0.30   | 0.40   | 0.50   | 0.60   | 0.70   | 0.80   | 0.90   |
| 8        | 0        | 0.4305   | 0.1678 | 0.1001 | 0.0576 | 0.0168 | 0.0039 | 0.0007 | 0.0001 | 0.0000 |        |
|          | 1        | 0.8131   | 0.5033 | 0.3671 | 0.2553 | 0.1064 | 0.0352 | 0.0085 | 0.0013 | 0.0001 |        |
|          | 2        | 0.9619   | 0.7969 | 0.6785 | 0.5518 | 0.3154 | 0.1445 | 0.0498 | 0.0113 | 0.0012 | 0.0000 |
|          | 3        | 0.9950   | 0.9437 | 0.8862 | 0.8059 | 0.5941 | 0.3633 | 0.1737 | 0.0580 | 0.0104 | 0.0004 |
|          | 4        | 0.9996   | 0.9896 | 0.9727 | 0.9420 | 0.8263 | 0.6367 | 0.4059 | 0.1941 | 0.0563 | 0.0050 |
|          | 5        | 1.0000   | 0.9988 | 0.9958 | 0.9887 | 0.9502 | 0.8555 | 0.6846 | 0.4482 | 0.2031 | 0.0381 |
|          | 6        |          | 0.9999 | 0.9996 | 0.9987 | 0.9915 | 0.9648 | 0.8936 | 0.7447 | 0.4967 | 0.1869 |
|          | 7        |          | 1.0000 | 1.0000 | 0.9999 | 0.9993 | 0.9961 | 0.9832 | 0.9424 | 0.8322 | 0.5695 |
|          | 8        |          |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 9        | 0        | 0.3874   | 0.1342 | 0.0751 | 0.0404 | 0.0101 | 0.0020 | 0.0003 | 0.0000 |        |        |
|          | 1        | 0.7748   | 0.4362 | 0.3003 | 0.1960 | 0.0705 | 0.0195 | 0.0038 | 0.0004 | 0.0000 |        |
|          | 2        | 0.9470   | 0.7382 | 0.6007 | 0.4628 | 0.2318 | 0.0898 | 0.0250 | 0.0043 | 0.0003 | 0.0000 |
|          | 3        | 0.9917   | 0.9144 | 0.8343 | 0.7297 | 0.4826 | 0.2539 | 0.0994 | 0.0253 | 0.0031 | 0.0001 |
|          | 4        | 0.9991   | 0.9804 | 0.9511 | 0.9012 | 0.7334 | 0.5000 | 0.2666 | 0.0988 | 0.0196 | 0.0009 |
|          | 5        | 0.9999   | 0.9969 | 0.9900 | 0.9747 | 0.9006 | 0.7461 | 0.5174 | 0.2703 | 0.0856 | 0.0083 |
|          | 6        | 1.0000   | 0.9997 | 0.9987 | 0.9957 | 0.9750 | 0.9102 | 0.7682 | 0.5372 | 0.2618 | 0.0530 |
|          | 7        |          | 1.0000 | 0.9999 | 0.9996 | 0.9962 | 0.9805 | 0.9295 | 0.8040 | 0.5638 | 0.2252 |
|          | 8        |          |        | 1.0000 | 1.0000 | 0.9997 | 0.9980 | 0.9899 | 0.9596 | 0.8658 | 0.6126 |
|          | 9        |          |        |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 10       | 0        | 0.3487   | 0.1074 | 0.0563 | 0.0282 | 0.0060 | 0.0010 | 0.0001 | 0.0000 |        |        |
|          | 1        | 0.7361   | 0.3758 | 0.2440 | 0.1493 | 0.0464 | 0.0107 | 0.0017 | 0.0001 | 0.0000 |        |
|          | 2        | 0.9298   | 0.6778 | 0.5256 | 0.3828 | 0.1673 | 0.0547 | 0.0123 | 0.0016 | 0.0001 |        |
|          | 3        | 0.9872   | 0.8791 | 0.7759 | 0.6496 | 0.3823 | 0.1719 | 0.0548 | 0.0106 | 0.0009 | 0.0000 |
|          | 4        | 0.9984   | 0.9672 | 0.9219 | 0.8497 | 0.6331 | 0.3770 | 0.1662 | 0.0473 | 0.0064 | 0.0001 |
|          | 5        | 0.9999   | 0.9936 | 0.9803 | 0.9527 | 0.8338 | 0.6230 | 0.3669 | 0.1503 | 0.0328 | 0.0016 |
|          | 6        | 1.0000   | 0.9991 | 0.9965 | 0.9894 | 0.9452 | 0.8281 | 0.6177 | 0.3504 | 0.1209 | 0.0128 |
|          | 7        |          | 0.9999 | 0.9996 | 0.9984 | 0.9877 | 0.9453 | 0.8327 | 0.6172 | 0.3222 | 0.0702 |
|          | 8        |          | 1.0000 | 1.0000 | 0.9999 | 0.9983 | 0.9893 | 0.9536 | 0.8507 | 0.6242 | 0.2639 |
|          | 9        |          |        |        | 1.0000 | 0.9999 | 0.9990 | 0.9940 | 0.9718 | 0.8926 | 0.6513 |
|          | 10       |          |        |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 11       | 0        | 0.3138   | 0.0859 | 0.0422 | 0.0198 | 0.0036 | 0.0005 | 0.0000 |        |        |        |
|          | 1        | 0.6974   | 0.3221 | 0.1971 | 0.1130 | 0.0302 | 0.0059 | 0.0007 | 0.0000 |        |        |
|          | 2        | 0.9104   | 0.6174 | 0.4552 | 0.3127 | 0.1189 | 0.0327 | 0.0059 | 0.0006 | 0.0000 |        |
|          | 3        | 0.9815   | 0.8389 | 0.7133 | 0.5696 | 0.2963 | 0.1133 | 0.0293 | 0.0043 | 0.0002 |        |
|          | 4        | 0.9972   | 0.9496 | 0.8854 | 0.7897 | 0.5328 | 0.2744 | 0.0994 | 0.0216 | 0.0020 | 0.0000 |
|          | 5        | 0.9997   | 0.9883 | 0.9657 | 0.9218 | 0.7535 | 0.5000 | 0.2465 | 0.0782 | 0.0117 | 0.0003 |
|          | 6        | 1.0000   | 0.9980 | 0.9924 | 0.9784 | 0.9006 | 0.7256 | 0.4672 | 0.2103 | 0.0504 | 0.0028 |
|          | 7        |          | 0.9998 | 0.9988 | 0.9957 | 0.9707 | 0.8867 | 0.7037 | 0.4304 | 0.1611 | 0.0185 |
|          | 8        |          | 1.0000 | 0.9999 | 0.9994 | 0.9941 | 0.9673 | 0.8811 | 0.6873 | 0.3826 | 0.0896 |
|          | 9        |          |        | 1.0000 | 1.0000 | 0.9993 | 0.9941 | 0.9698 | 0.8870 | 0.6779 | 0.3026 |
|          | 10       |          |        |        |        | 1.0000 | 0.9995 | 0.9964 | 0.9802 | 0.9141 | 0.6862 |
|          | 11       |          |        |        |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

**Table A.1** (continued) Binomial Probability Sums  $\sum_{x=0}^r b(x; n, p)$

| <i>n</i> | <i>r</i> | <i>p</i> |        |        |        |        |        |        |        |        |        |
|----------|----------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|          |          | 0.10     | 0.20   | 0.25   | 0.30   | 0.40   | 0.50   | 0.60   | 0.70   | 0.80   | 0.90   |
| 12       | 0        | 0.2824   | 0.0687 | 0.0317 | 0.0138 | 0.0022 | 0.0002 | 0.0000 |        |        |        |
|          | 1        | 0.6590   | 0.2749 | 0.1584 | 0.0850 | 0.0196 | 0.0032 | 0.0003 | 0.0000 |        |        |
|          | 2        | 0.8891   | 0.5583 | 0.3907 | 0.2528 | 0.0834 | 0.0193 | 0.0028 | 0.0002 | 0.0000 |        |
|          | 3        | 0.9744   | 0.7946 | 0.6488 | 0.4925 | 0.2253 | 0.0730 | 0.0153 | 0.0017 | 0.0001 |        |
|          | 4        | 0.9957   | 0.9274 | 0.8424 | 0.7237 | 0.4382 | 0.1938 | 0.0573 | 0.0095 | 0.0006 | 0.0000 |
|          | 5        | 0.9995   | 0.9806 | 0.9456 | 0.8822 | 0.6652 | 0.3872 | 0.1582 | 0.0386 | 0.0039 | 0.0001 |
|          | 6        | 0.9999   | 0.9961 | 0.9857 | 0.9614 | 0.8418 | 0.6128 | 0.3348 | 0.1178 | 0.0194 | 0.0005 |
|          | 7        | 1.0000   | 0.9994 | 0.9972 | 0.9905 | 0.9427 | 0.8062 | 0.5618 | 0.2763 | 0.0726 | 0.0043 |
|          | 8        |          | 0.9999 | 0.9996 | 0.9983 | 0.9847 | 0.9270 | 0.7747 | 0.5075 | 0.2054 | 0.0256 |
|          | 9        |          | 1.0000 | 1.0000 | 0.9998 | 0.9972 | 0.9807 | 0.9166 | 0.7472 | 0.4417 | 0.1109 |
|          | 10       |          |        |        | 1.0000 | 0.9997 | 0.9968 | 0.9804 | 0.9150 | 0.7251 | 0.3410 |
|          | 11       |          |        |        |        | 1.0000 | 0.9998 | 0.9978 | 0.9862 | 0.9313 | 0.7176 |
|          | 12       |          |        |        |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 13       | 0        | 0.2542   | 0.0550 | 0.0238 | 0.0097 | 0.0013 | 0.0001 | 0.0000 |        |        |        |
|          | 1        | 0.6213   | 0.2336 | 0.1267 | 0.0637 | 0.0126 | 0.0017 | 0.0001 | 0.0000 |        |        |
|          | 2        | 0.8661   | 0.5017 | 0.3326 | 0.2025 | 0.0579 | 0.0112 | 0.0013 | 0.0001 |        |        |
|          | 3        | 0.9658   | 0.7473 | 0.5843 | 0.4206 | 0.1686 | 0.0461 | 0.0078 | 0.0007 | 0.0000 |        |
|          | 4        | 0.9935   | 0.9009 | 0.7940 | 0.6543 | 0.3530 | 0.1334 | 0.0321 | 0.0040 | 0.0002 |        |
|          | 5        | 0.9991   | 0.9700 | 0.9198 | 0.8346 | 0.5744 | 0.2905 | 0.0977 | 0.0182 | 0.0012 | 0.0000 |
|          | 6        | 0.9999   | 0.9930 | 0.9757 | 0.9376 | 0.7712 | 0.5000 | 0.2288 | 0.0624 | 0.0070 | 0.0001 |
|          | 7        | 1.0000   | 0.9988 | 0.9944 | 0.9818 | 0.9023 | 0.7095 | 0.4256 | 0.1654 | 0.0300 | 0.0009 |
|          | 8        |          | 0.9998 | 0.9990 | 0.9960 | 0.9679 | 0.8666 | 0.6470 | 0.3457 | 0.0991 | 0.0065 |
|          | 9        |          | 1.0000 | 0.9999 | 0.9993 | 0.9922 | 0.9539 | 0.8314 | 0.5794 | 0.2527 | 0.0342 |
|          | 10       |          |        | 1.0000 | 0.9999 | 0.9987 | 0.9888 | 0.9421 | 0.7975 | 0.4983 | 0.1339 |
|          | 11       |          |        |        | 1.0000 | 0.9999 | 0.9983 | 0.9874 | 0.9363 | 0.7664 | 0.3787 |
|          | 12       |          |        |        |        | 1.0000 | 0.9999 | 0.9987 | 0.9903 | 0.9450 | 0.7458 |
|          | 13       |          |        |        |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 14       | 0        | 0.2288   | 0.0440 | 0.0178 | 0.0068 | 0.0008 | 0.0001 | 0.0000 |        |        |        |
|          | 1        | 0.5846   | 0.1979 | 0.1010 | 0.0475 | 0.0081 | 0.0009 | 0.0001 |        |        |        |
|          | 2        | 0.8416   | 0.4481 | 0.2811 | 0.1608 | 0.0398 | 0.0065 | 0.0006 | 0.0000 |        |        |
|          | 3        | 0.9559   | 0.6982 | 0.5213 | 0.3552 | 0.1243 | 0.0287 | 0.0039 | 0.0002 |        |        |
|          | 4        | 0.9908   | 0.8702 | 0.7415 | 0.5842 | 0.2793 | 0.0898 | 0.0175 | 0.0017 | 0.0000 |        |
|          | 5        | 0.9985   | 0.9561 | 0.8883 | 0.7805 | 0.4859 | 0.2120 | 0.0583 | 0.0083 | 0.0004 |        |
|          | 6        | 0.9998   | 0.9884 | 0.9617 | 0.9067 | 0.6925 | 0.3953 | 0.1501 | 0.0315 | 0.0024 | 0.0000 |
|          | 7        | 1.0000   | 0.9976 | 0.9897 | 0.9685 | 0.8499 | 0.6047 | 0.3075 | 0.0933 | 0.0116 | 0.0002 |
|          | 8        |          | 0.9996 | 0.9978 | 0.9917 | 0.9417 | 0.7880 | 0.5141 | 0.2195 | 0.0439 | 0.0015 |
|          | 9        |          | 1.0000 | 0.9997 | 0.9983 | 0.9825 | 0.9102 | 0.7207 | 0.4158 | 0.1298 | 0.0092 |
|          | 10       |          |        | 1.0000 | 0.9998 | 0.9961 | 0.9713 | 0.8757 | 0.6448 | 0.3018 | 0.0441 |
|          | 11       |          |        |        | 1.0000 | 0.9994 | 0.9935 | 0.9602 | 0.8392 | 0.5519 | 0.1584 |
|          | 12       |          |        |        |        | 0.9999 | 0.9991 | 0.9919 | 0.9525 | 0.8021 | 0.4154 |
|          | 13       |          |        |        |        | 1.0000 | 0.9999 | 0.9992 | 0.9932 | 0.9560 | 0.7712 |
|          | 14       |          |        |        |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

Table A.1 (continued) Binomial Probability Sums  $\sum_{x=0}^r b(x; n, p)$ 

| <i>n</i> | <i>r</i> | <i>p</i> |        |        |        |        |        |        |        |        |        |
|----------|----------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|          |          | 0.10     | 0.20   | 0.25   | 0.30   | 0.40   | 0.50   | 0.60   | 0.70   | 0.80   | 0.90   |
| 15       | 0        | 0.2059   | 0.0352 | 0.0134 | 0.0047 | 0.0005 | 0.0000 |        |        |        |        |
|          | 1        | 0.5490   | 0.1671 | 0.0802 | 0.0353 | 0.0052 | 0.0005 | 0.0000 |        |        |        |
|          | 2        | 0.8159   | 0.3980 | 0.2361 | 0.1268 | 0.0271 | 0.0037 | 0.0003 | 0.0000 |        |        |
|          | 3        | 0.9444   | 0.6482 | 0.4613 | 0.2969 | 0.0905 | 0.0176 | 0.0019 | 0.0001 |        |        |
|          | 4        | 0.9873   | 0.8358 | 0.6865 | 0.5155 | 0.2173 | 0.0592 | 0.0093 | 0.0007 | 0.0000 |        |
|          | 5        | 0.9978   | 0.9389 | 0.8516 | 0.7216 | 0.4032 | 0.1509 | 0.0338 | 0.0037 | 0.0001 |        |
|          | 6        | 0.9997   | 0.9819 | 0.9434 | 0.8689 | 0.6098 | 0.3036 | 0.0950 | 0.0152 | 0.0008 |        |
|          | 7        | 1.0000   | 0.9958 | 0.9827 | 0.9500 | 0.7869 | 0.5000 | 0.2131 | 0.0500 | 0.0042 | 0.0000 |
|          | 8        |          | 0.9992 | 0.9958 | 0.9848 | 0.9050 | 0.6964 | 0.3902 | 0.1311 | 0.0181 | 0.0003 |
|          | 9        |          | 0.9999 | 0.9992 | 0.9963 | 0.9662 | 0.8491 | 0.5968 | 0.2784 | 0.0611 | 0.0022 |
|          | 10       |          | 1.0000 | 0.9999 | 0.9993 | 0.9907 | 0.9408 | 0.7827 | 0.4845 | 0.1642 | 0.0127 |
|          | 11       |          |        | 1.0000 | 0.9999 | 0.9981 | 0.9824 | 0.9095 | 0.7031 | 0.3518 | 0.0556 |
|          | 12       |          |        |        | 1.0000 | 0.9997 | 0.9963 | 0.9729 | 0.8732 | 0.6020 | 0.1841 |
|          | 13       |          |        |        |        | 1.0000 | 0.9995 | 0.9948 | 0.9647 | 0.8329 | 0.4510 |
|          | 14       |          |        |        |        |        | 1.0000 | 0.9995 | 0.9953 | 0.9648 | 0.7941 |
|          | 15       |          |        |        |        |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 16       | 0        | 0.1853   | 0.0281 | 0.0100 | 0.0033 | 0.0003 | 0.0000 |        |        |        |        |
|          | 1        | 0.5147   | 0.1407 | 0.0635 | 0.0261 | 0.0033 | 0.0003 | 0.0000 |        |        |        |
|          | 2        | 0.7892   | 0.3518 | 0.1971 | 0.0994 | 0.0183 | 0.0021 | 0.0001 |        |        |        |
|          | 3        | 0.9316   | 0.5981 | 0.4050 | 0.2459 | 0.0651 | 0.0106 | 0.0009 | 0.0000 |        |        |
|          | 4        | 0.9830   | 0.7982 | 0.6302 | 0.4499 | 0.1666 | 0.0384 | 0.0049 | 0.0003 |        |        |
|          | 5        | 0.9967   | 0.9183 | 0.8103 | 0.6598 | 0.3288 | 0.1051 | 0.0191 | 0.0016 | 0.0000 |        |
|          | 6        | 0.9995   | 0.9733 | 0.9204 | 0.8247 | 0.5272 | 0.2272 | 0.0583 | 0.0071 | 0.0002 |        |
|          | 7        | 0.9999   | 0.9930 | 0.9729 | 0.9256 | 0.7161 | 0.4018 | 0.1423 | 0.0257 | 0.0015 | 0.0000 |
|          | 8        | 1.0000   | 0.9985 | 0.9925 | 0.9743 | 0.8577 | 0.5982 | 0.2839 | 0.0744 | 0.0070 | 0.0001 |
|          | 9        |          | 0.9998 | 0.9984 | 0.9929 | 0.9417 | 0.7728 | 0.4728 | 0.1753 | 0.0267 | 0.0005 |
|          | 10       |          | 1.0000 | 0.9997 | 0.9984 | 0.9809 | 0.8949 | 0.6712 | 0.3402 | 0.0817 | 0.0033 |
|          | 11       |          |        | 1.0000 | 0.9997 | 0.9951 | 0.9616 | 0.8334 | 0.5501 | 0.2018 | 0.0170 |
|          | 12       |          |        |        | 1.0000 | 0.9991 | 0.9894 | 0.9349 | 0.7541 | 0.4019 | 0.0684 |
|          | 13       |          |        |        |        | 0.9999 | 0.9979 | 0.9817 | 0.9006 | 0.6482 | 0.2108 |
|          | 14       |          |        |        |        | 1.0000 | 0.9997 | 0.9967 | 0.9739 | 0.8593 | 0.4853 |
|          | 15       |          |        |        |        |        | 1.0000 | 0.9997 | 0.9967 | 0.9719 | 0.8147 |
|          | 16       |          |        |        |        |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

**Table A.1** (continued) Binomial Probability Sums  $\sum_{x=0}^r b(x; n, p)$

| <i>n</i> | <i>r</i> | <i>p</i> |        |        |        |        |        |        |        |        |        |
|----------|----------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|          |          | 0.10     | 0.20   | 0.25   | 0.30   | 0.40   | 0.50   | 0.60   | 0.70   | 0.80   | 0.90   |
| 17       | 0        | 0.1668   | 0.0225 | 0.0075 | 0.0023 | 0.0002 | 0.0000 |        |        |        |        |
|          | 1        | 0.4818   | 0.1182 | 0.0501 | 0.0193 | 0.0021 | 0.0001 | 0.0000 |        |        |        |
|          | 2        | 0.7618   | 0.3096 | 0.1637 | 0.0774 | 0.0123 | 0.0012 | 0.0001 |        |        |        |
|          | 3        | 0.9174   | 0.5489 | 0.3530 | 0.2019 | 0.0464 | 0.0064 | 0.0005 | 0.0000 |        |        |
|          | 4        | 0.9779   | 0.7582 | 0.5739 | 0.3887 | 0.1260 | 0.0245 | 0.0025 | 0.0001 |        |        |
|          | 5        | 0.9953   | 0.8943 | 0.7653 | 0.5968 | 0.2639 | 0.0717 | 0.0106 | 0.0007 | 0.0000 |        |
|          | 6        | 0.9992   | 0.9623 | 0.8929 | 0.7752 | 0.4478 | 0.1662 | 0.0348 | 0.0032 | 0.0001 |        |
|          | 7        | 0.9999   | 0.9891 | 0.9598 | 0.8954 | 0.6405 | 0.3145 | 0.0919 | 0.0127 | 0.0005 |        |
|          | 8        | 1.0000   | 0.9974 | 0.9876 | 0.9597 | 0.8011 | 0.5000 | 0.1989 | 0.0403 | 0.0026 | 0.0000 |
|          | 9        |          | 0.9995 | 0.9969 | 0.9873 | 0.9081 | 0.6855 | 0.3595 | 0.1046 | 0.0109 | 0.0001 |
|          | 10       |          | 0.9999 | 0.9994 | 0.9968 | 0.9652 | 0.8338 | 0.5522 | 0.2248 | 0.0377 | 0.0008 |
|          | 11       |          | 1.0000 | 0.9999 | 0.9993 | 0.9894 | 0.9283 | 0.7361 | 0.4032 | 0.1057 | 0.0047 |
|          | 12       |          |        | 1.0000 | 0.9999 | 0.9975 | 0.9755 | 0.8740 | 0.6113 | 0.2418 | 0.0221 |
|          | 13       |          |        |        | 1.0000 | 0.9995 | 0.9936 | 0.9536 | 0.7981 | 0.4511 | 0.0826 |
|          | 14       |          |        |        |        | 0.9999 | 0.9988 | 0.9877 | 0.9226 | 0.6904 | 0.2382 |
|          | 15       |          |        |        |        | 1.0000 | 0.9999 | 0.9979 | 0.9807 | 0.8818 | 0.5182 |
|          | 16       |          |        |        |        |        | 1.0000 | 0.9998 | 0.9977 | 0.9775 | 0.8332 |
|          | 17       |          |        |        |        |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 18       | 0        | 0.1501   | 0.0180 | 0.0056 | 0.0016 | 0.0001 | 0.0000 |        |        |        |        |
|          | 1        | 0.4503   | 0.0991 | 0.0395 | 0.0142 | 0.0013 | 0.0001 |        |        |        |        |
|          | 2        | 0.7338   | 0.2713 | 0.1353 | 0.0600 | 0.0082 | 0.0007 | 0.0000 |        |        |        |
|          | 3        | 0.9018   | 0.5010 | 0.3057 | 0.1646 | 0.0328 | 0.0038 | 0.0002 |        |        |        |
|          | 4        | 0.9718   | 0.7164 | 0.5187 | 0.3327 | 0.0942 | 0.0154 | 0.0013 | 0.0000 |        |        |
|          | 5        | 0.9936   | 0.8671 | 0.7175 | 0.5344 | 0.2088 | 0.0481 | 0.0058 | 0.0003 |        |        |
|          | 6        | 0.9988   | 0.9487 | 0.8610 | 0.7217 | 0.3743 | 0.1189 | 0.0203 | 0.0014 | 0.0000 |        |
|          | 7        | 0.9998   | 0.9837 | 0.9431 | 0.8593 | 0.5634 | 0.2403 | 0.0576 | 0.0061 | 0.0002 |        |
|          | 8        | 1.0000   | 0.9957 | 0.9807 | 0.9404 | 0.7368 | 0.4073 | 0.1347 | 0.0210 | 0.0009 |        |
|          | 9        |          | 0.9991 | 0.9946 | 0.9790 | 0.8653 | 0.5927 | 0.2632 | 0.0596 | 0.0043 | 0.0000 |
|          | 10       |          | 0.9998 | 0.9988 | 0.9939 | 0.9424 | 0.7597 | 0.4366 | 0.1407 | 0.0163 | 0.0002 |
|          | 11       |          | 1.0000 | 0.9998 | 0.9986 | 0.9797 | 0.8811 | 0.6257 | 0.2783 | 0.0513 | 0.0012 |
|          | 12       |          |        | 1.0000 | 0.9997 | 0.9942 | 0.9519 | 0.7912 | 0.4656 | 0.1329 | 0.0064 |
|          | 13       |          |        |        | 1.0000 | 0.9987 | 0.9846 | 0.9058 | 0.6673 | 0.2836 | 0.0282 |
|          | 14       |          |        |        |        | 0.9998 | 0.9962 | 0.9672 | 0.8354 | 0.4990 | 0.0982 |
|          | 15       |          |        |        |        | 1.0000 | 0.9993 | 0.9918 | 0.9400 | 0.7287 | 0.2662 |
|          | 16       |          |        |        |        |        | 0.9999 | 0.9987 | 0.9858 | 0.9009 | 0.5497 |
|          | 17       |          |        |        |        |        | 1.0000 | 0.9999 | 0.9984 | 0.9820 | 0.8499 |
|          | 18       |          |        |        |        |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

Table A.1 (continued) Binomial Probability Sums  $\sum_{x=0}^r b(x; n, p)$ 

| <i>n</i> | <i>r</i> | <i>p</i> |        |        |        |        |        |        |        |        |        |
|----------|----------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
|          |          | 0.10     | 0.20   | 0.25   | 0.30   | 0.40   | 0.50   | 0.60   | 0.70   | 0.80   | 0.90   |
| 19       | 0        | 0.1351   | 0.0144 | 0.0042 | 0.0011 | 0.0001 |        |        |        |        |        |
|          | 1        | 0.4203   | 0.0829 | 0.0310 | 0.0104 | 0.0008 | 0.0000 |        |        |        |        |
|          | 2        | 0.7054   | 0.2369 | 0.1113 | 0.0462 | 0.0055 | 0.0004 | 0.0000 |        |        |        |
|          | 3        | 0.8850   | 0.4551 | 0.2631 | 0.1332 | 0.0230 | 0.0022 | 0.0001 |        |        |        |
|          | 4        | 0.9648   | 0.6733 | 0.4654 | 0.2822 | 0.0696 | 0.0096 | 0.0006 | 0.0000 |        |        |
|          | 5        | 0.9914   | 0.8369 | 0.6678 | 0.4739 | 0.1629 | 0.0318 | 0.0031 | 0.0001 |        |        |
|          | 6        | 0.9983   | 0.9324 | 0.8251 | 0.6655 | 0.3081 | 0.0835 | 0.0116 | 0.0006 |        |        |
|          | 7        | 0.9997   | 0.9767 | 0.9225 | 0.8180 | 0.4878 | 0.1796 | 0.0352 | 0.0028 | 0.0000 |        |
|          | 8        | 1.0000   | 0.9933 | 0.9713 | 0.9161 | 0.6675 | 0.3238 | 0.0885 | 0.0105 | 0.0003 |        |
|          | 9        |          | 0.9984 | 0.9911 | 0.9674 | 0.8139 | 0.5000 | 0.1861 | 0.0326 | 0.0016 |        |
|          | 10       |          | 0.9997 | 0.9977 | 0.9895 | 0.9115 | 0.6762 | 0.3325 | 0.0839 | 0.0067 | 0.0000 |
|          | 11       |          | 1.0000 | 0.9995 | 0.9972 | 0.9648 | 0.8204 | 0.5122 | 0.1820 | 0.0233 | 0.0003 |
|          | 12       |          |        | 0.9999 | 0.9994 | 0.9884 | 0.9165 | 0.6919 | 0.3345 | 0.0676 | 0.0017 |
|          | 13       |          |        | 1.0000 | 0.9999 | 0.9969 | 0.9682 | 0.8371 | 0.5261 | 0.1631 | 0.0086 |
|          | 14       |          |        |        | 1.0000 | 0.9994 | 0.9904 | 0.9304 | 0.7178 | 0.3267 | 0.0352 |
|          | 15       |          |        |        |        | 0.9999 | 0.9978 | 0.9770 | 0.8668 | 0.5449 | 0.1150 |
|          | 16       |          |        |        |        | 1.0000 | 0.9996 | 0.9945 | 0.9538 | 0.7631 | 0.2946 |
|          | 17       |          |        |        |        |        | 1.0000 | 0.9992 | 0.9896 | 0.9171 | 0.5797 |
|          | 18       |          |        |        |        |        |        | 0.9999 | 0.9989 | 0.9856 | 0.8649 |
|          | 19       |          |        |        |        |        |        | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 20       | 0        | 0.1216   | 0.0115 | 0.0032 | 0.0008 | 0.0000 |        |        |        |        |        |
|          | 1        | 0.3917   | 0.0692 | 0.0243 | 0.0076 | 0.0005 | 0.0000 |        |        |        |        |
|          | 2        | 0.6769   | 0.2061 | 0.0913 | 0.0355 | 0.0036 | 0.0002 |        |        |        |        |
|          | 3        | 0.8670   | 0.4114 | 0.2252 | 0.1071 | 0.0160 | 0.0013 | 0.0000 |        |        |        |
|          | 4        | 0.9568   | 0.6296 | 0.4148 | 0.2375 | 0.0510 | 0.0059 | 0.0003 |        |        |        |
|          | 5        | 0.9887   | 0.8042 | 0.6172 | 0.4164 | 0.1256 | 0.0207 | 0.0016 | 0.0000 |        |        |
|          | 6        | 0.9976   | 0.9133 | 0.7858 | 0.6080 | 0.2500 | 0.0577 | 0.0065 | 0.0003 |        |        |
|          | 7        | 0.9996   | 0.9679 | 0.8982 | 0.7723 | 0.4159 | 0.1316 | 0.0210 | 0.0013 | 0.0000 |        |
|          | 8        | 0.9999   | 0.9900 | 0.9591 | 0.8867 | 0.5956 | 0.2517 | 0.0565 | 0.0051 | 0.0001 |        |
|          | 9        | 1.0000   | 0.9974 | 0.9861 | 0.9520 | 0.7553 | 0.4119 | 0.1275 | 0.0171 | 0.0006 |        |
|          | 10       |          | 0.9994 | 0.9961 | 0.9829 | 0.8725 | 0.5881 | 0.2447 | 0.0480 | 0.0026 | 0.0000 |
|          | 11       |          | 0.9999 | 0.9991 | 0.9949 | 0.9435 | 0.7483 | 0.4044 | 0.1133 | 0.0100 | 0.0001 |
|          | 12       |          | 1.0000 | 0.9998 | 0.9987 | 0.9790 | 0.8684 | 0.5841 | 0.2277 | 0.0321 | 0.0004 |
|          | 13       |          |        | 1.0000 | 0.9997 | 0.9935 | 0.9423 | 0.7500 | 0.3920 | 0.0867 | 0.0024 |
|          | 14       |          |        |        | 1.0000 | 0.9984 | 0.9793 | 0.8744 | 0.5836 | 0.1958 | 0.0113 |
|          | 15       |          |        |        |        | 0.9997 | 0.9941 | 0.9490 | 0.7625 | 0.3704 | 0.0432 |
|          | 16       |          |        |        |        | 1.0000 | 0.9987 | 0.9840 | 0.8929 | 0.5886 | 0.1330 |
|          | 17       |          |        |        |        |        | 0.9998 | 0.9964 | 0.9645 | 0.7939 | 0.3231 |
|          | 18       |          |        |        |        |        | 1.0000 | 0.9995 | 0.9924 | 0.9308 | 0.6083 |
|          | 19       |          |        |        |        |        |        | 1.0000 | 0.9992 | 0.9885 | 0.8784 |
|          | 20       |          |        |        |        |        |        |        | 1.0000 | 1.0000 | 1.0000 |



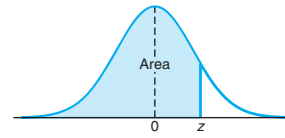
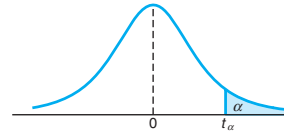


Table A.3 Areas under the Normal Curve

| $z$  | .00    | .01    | .02    | .03    | .04    | .05    | .06    | .07    | .08    | .09    |
|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| −3.4 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| −3.3 | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| −3.2 | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| −3.1 | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| −3.0 | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| −2.9 | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| −2.8 | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| −2.7 | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| −2.6 | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| −2.5 | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| −2.4 | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| −2.3 | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| −2.2 | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| −2.1 | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| −2.0 | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| −1.9 | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| −1.8 | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| −1.7 | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| −1.6 | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| −1.5 | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| −1.4 | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| −1.3 | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| −1.2 | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| −1.1 | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| −1.0 | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| −0.9 | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| −0.8 | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| −0.7 | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| −0.6 | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| −0.5 | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| −0.4 | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| −0.3 | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| −0.2 | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| −0.1 | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| −0.0 | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |



**Table A.4** Critical Values of the  $t$ -Distribution

| $v$      | $\alpha$ |       |       |       |       |       |        |
|----------|----------|-------|-------|-------|-------|-------|--------|
|          | 0.40     | 0.30  | 0.20  | 0.15  | 0.10  | 0.05  | 0.025  |
| 1        | 0.325    | 0.727 | 1.376 | 1.963 | 3.078 | 6.314 | 12.706 |
| 2        | 0.289    | 0.617 | 1.061 | 1.386 | 1.886 | 2.920 | 4.303  |
| 3        | 0.277    | 0.584 | 0.978 | 1.250 | 1.638 | 2.353 | 3.182  |
| 4        | 0.271    | 0.569 | 0.941 | 1.190 | 1.533 | 2.132 | 2.776  |
| 5        | 0.267    | 0.559 | 0.920 | 1.156 | 1.476 | 2.015 | 2.571  |
| 6        | 0.265    | 0.553 | 0.906 | 1.134 | 1.440 | 1.943 | 2.447  |
| 7        | 0.263    | 0.549 | 0.896 | 1.119 | 1.415 | 1.895 | 2.365  |
| 8        | 0.262    | 0.546 | 0.889 | 1.108 | 1.397 | 1.860 | 2.306  |
| 9        | 0.261    | 0.543 | 0.883 | 1.100 | 1.383 | 1.833 | 2.262  |
| 10       | 0.260    | 0.542 | 0.879 | 1.093 | 1.372 | 1.812 | 2.228  |
| 11       | 0.260    | 0.540 | 0.876 | 1.088 | 1.363 | 1.796 | 2.201  |
| 12       | 0.259    | 0.539 | 0.873 | 1.083 | 1.356 | 1.782 | 2.179  |
| 13       | 0.259    | 0.538 | 0.870 | 1.079 | 1.350 | 1.771 | 2.160  |
| 14       | 0.258    | 0.537 | 0.868 | 1.076 | 1.345 | 1.761 | 2.145  |
| 15       | 0.258    | 0.536 | 0.866 | 1.074 | 1.341 | 1.753 | 2.131  |
| 16       | 0.258    | 0.535 | 0.865 | 1.071 | 1.337 | 1.746 | 2.120  |
| 17       | 0.257    | 0.534 | 0.863 | 1.069 | 1.333 | 1.740 | 2.110  |
| 18       | 0.257    | 0.534 | 0.862 | 1.067 | 1.330 | 1.734 | 2.101  |
| 19       | 0.257    | 0.533 | 0.861 | 1.066 | 1.328 | 1.729 | 2.093  |
| 20       | 0.257    | 0.533 | 0.860 | 1.064 | 1.325 | 1.725 | 2.086  |
| 21       | 0.257    | 0.532 | 0.859 | 1.063 | 1.323 | 1.721 | 2.080  |
| 22       | 0.256    | 0.532 | 0.858 | 1.061 | 1.321 | 1.717 | 2.074  |
| 23       | 0.256    | 0.532 | 0.858 | 1.060 | 1.319 | 1.714 | 2.069  |
| 24       | 0.256    | 0.531 | 0.857 | 1.059 | 1.318 | 1.711 | 2.064  |
| 25       | 0.256    | 0.531 | 0.856 | 1.058 | 1.316 | 1.708 | 2.060  |
| 26       | 0.256    | 0.531 | 0.856 | 1.058 | 1.315 | 1.706 | 2.056  |
| 27       | 0.256    | 0.531 | 0.855 | 1.057 | 1.314 | 1.703 | 2.052  |
| 28       | 0.256    | 0.530 | 0.855 | 1.056 | 1.313 | 1.701 | 2.048  |
| 29       | 0.256    | 0.530 | 0.854 | 1.055 | 1.311 | 1.699 | 2.045  |
| 30       | 0.256    | 0.530 | 0.854 | 1.055 | 1.310 | 1.697 | 2.042  |
| 40       | 0.255    | 0.529 | 0.851 | 1.050 | 1.303 | 1.684 | 2.021  |
| 60       | 0.254    | 0.527 | 0.848 | 1.045 | 1.296 | 1.671 | 2.000  |
| 120      | 0.254    | 0.526 | 0.845 | 1.041 | 1.289 | 1.658 | 1.980  |
| $\infty$ | 0.253    | 0.524 | 0.842 | 1.036 | 1.282 | 1.645 | 1.960  |

**Control Chart Constants**

| $n$ | $A_3$ | $B_3$ | $B_4$ | $A_2$ | $D_3$ | $D_4$ |
|-----|-------|-------|-------|-------|-------|-------|
| 4   | 1.63  | 0     | 2.27  | 0.73  | 0     | 2.28  |
| 5   | 1.43  | 0     | 2.09  | 0.58  | 0     | 2.11  |
| 6   | 1.29  | 0.03  | 1.97  | 0.48  | 0     | 2.00  |
| 8   | 1.10  | 0.185 | 1.815 | 0.37  | 0.14  | 1.86  |
| 10  | 0.98  | 0.28  | 1.72  | 0.31  | 0.22  | 1.78  |
| 20  | 0.68  | 0.51  | 1.49  | 0.18  | 0.41  | 1.59  |

| Sample Size = n | $A_2$ | $A_3$ |
|-----------------|-------|-------|
| 2               | 1.880 | 2.659 |
| 3               | 1.023 | 1.954 |
| 4               | 0.729 | 1.628 |
| 5               | 0.577 | 1.427 |
| 6               | 0.483 | 1.287 |
| 7               | 0.419 | 1.182 |
| 8               | 0.373 | 1.099 |
| 9               | 0.337 | 1.032 |
| 10              | 0.308 | 0.975 |
| 11              | 0.285 | 0.927 |
| 12              | 0.266 | 0.886 |
| 13              | 0.249 | 0.850 |
| 14              | 0.235 | 0.817 |
| 15              | 0.223 | 0.789 |
| 16              | 0.212 | 0.763 |
| 17              | 0.203 | 0.739 |
| 18              | 0.194 | 0.718 |
| 19              | 0.187 | 0.698 |
| 20              | 0.180 | 0.680 |
| 21              | 0.173 | 0.663 |
| 22              | 0.167 | 0.647 |
| 23              | 0.162 | 0.633 |
| 24              | 0.157 | 0.619 |
| 25              | 0.153 | 0.606 |