

MAT 2377
Winter 2020
Final Exam
April 20, 2020
Time Limit: 3 Hours

Final Examination
Student: BLECHNER R
Version: 91411
Start Time: 2PM EST

This exam contains 19 pages (including this cover page and tables) and 32 questions.

Read carefully: if you submit answers for this online examination, you promise and certify that

- all work done during the assigned exam period will be done entirely by yourself, with no help from others;
- you will not communicate with anybody except the professor during the exam, for exam- and course-related questions;
- you will not consult any people, sources, or writings other than the course textbook, your self-made review sheets, your course notes, your prepared answers, the course documents made available on Brightspace, and the exam itself;
- you will not provide information related to the exam's contents to other people until 24 hours after the exam is over.

Obviously, we do not have a way to monitor your adherence to these instructions (nor do we really wish to look for one); we will be asking, instead, for your word of honour that you will follow the directives and work on this examination alone.

Multiple choice questions and true or false question are worth 1 mark each; short answer questions, 2 marks. There are no part marks. For each question, record your answer in the corresponding space on the answer sheet that has been provided separately (save your file regularly). For multiple choice questions, select the closest answer among the options, when appropriate; for short answer questions, provide an answer to at least 2 decimals.

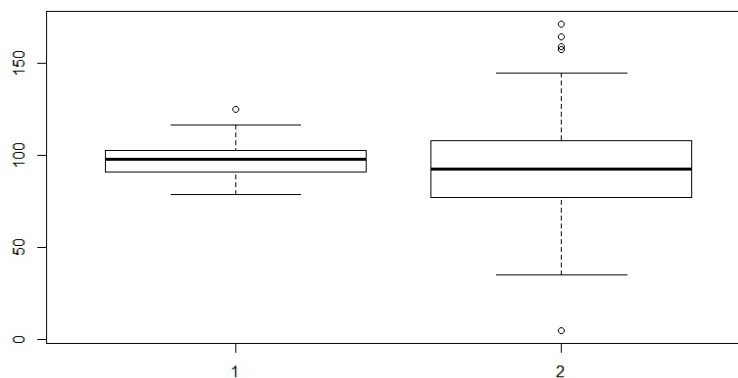
IMPORTANT: every student has been assigned a different exam (based on the last 5 digits of your student number). You need to answer the questions of the final exam that has been specifically assigned to you.

When you have answered all the questions and you are ready to upload the answers to Brightspace, save your answers to the file **Answer_Sheet_91411.pdf**. We suggest that you print and scan or take a screenshot (with time stamp) of your completed answer sheet before uploading to Brightspace (see the appropriate location in the “Assignment” section where you uploaded your 3rd assignment), and that you keep a copy of these documents (with time stamp). Should Brightspace's upload functionality not work when you are ready to submit, wait a little bit and try again; as a last recourse, please email your answer sheet with saved answers to pboily@uottawa.ca if you are registered in section A or iyadegar@uottawa.ca if you are registered in section B.

No worries. You've got this.

1. **Multiple Choice Question**

Consider the following boxplots:



It appears that the dispersion of

- A. the first sample is the largest
- B. the second sample is the largest
- C. the first and second sample is the same
- D. both samples cannot be compared
- E. none of the preceding

2. **Multiple Choice Question**

Consider the following random sample: 16, 13, 10, 17, 16, 14, 5, 12, 15, 8. Calculate the first quartile (choose the best answer).

- A. 9.5 B. 7.5 C. 10.5 D. 0.25 E. 11

3. **Multiple Choice Question**

Suppose that $X_1, \dots, X_{81}$ are independent random variables with probability density function $f_X(x) = 0.5 \exp(-x/2)$, $x > 0$. Approximate $P(X_1 + \dots + X_{81} > 170)$.

- A. 0.67 B. 0.16 C. 0.33 D. 0.95 E. none of the preceding

4. **Multiple Choice Question**

An engineer measures the weights (in kilograms) of steel pieces. They would like to test $H_0 : \mu = 5$ against $H_1 : \mu > 5$. The weights follow a normal distribution with variance 16. Using a sample of size $n = 25$, the engineer decides to reject H_0 if $\bar{x} > 6$. Assuming that the true population mean is 5.2, determine the probability of committing an error of Type II error.

- A. 0.8413 B. 0.05 C. 0.9332 D. 0.8943 E. none of the preceding

5. Multiple Choice Question

A machine produces cylindrical metal pieces. A random sample of the pieces yields diameters

1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.04, 1.03, 1.01.

Determine a 99% confidence interval for the mean diameter. You may assume the diameters are normally distributed.

- A. [0.989, 1.022] B. [0.983, 1.035] C. [0.991, 1.034] D. [0.987, 0.024]
E. none of the above

6. Multiple Choice Question

Let $X_1, \dots, X_7$ be a random sample from a population with mean μ and variance σ^2 . Consider the following estimators for μ :

$$\hat{\Theta}_1 = \frac{X_1 + \dots + X_7}{7}, \quad \hat{\Theta}_2 = \frac{2X_1 - X_6 + \dots + X_4}{2}.$$

Are these estimators unbiased (i.e. is their expectation equal to μ)?

- A. Both estimators are unbiased. B. Both estimators are biased.
C. Only the second is unbiased. D. Only the first is unbiased.
E. Insufficient information.

7. Multiple Choice Question

Previous experience has shown that the breaking strength of the fabric used in a certain brand of drapes is normally distributed with a standard deviation of 2 pounds per square inch. A random sample of 25 specimens is examined to reveal an average breaking strength of $\bar{x} = 98$ pounds per square inch. Determine the p -value required to test the hypothesis that the true mean is not 97.

- A. 0.067 B. 0.012 C. 0.13 D. 0.006 E. none of the preceding

8. Multiple Choice Question

Trucks arrive at a loading/unloading station according to a Poisson process with a rate of 2 trucks per hour. A truck just arrived at the station. A station employee would like to know if he has enough time to take a break. Let the random variable T represent the waiting time (in hours) until the next truck arrival. What is the probability that the employee has 15 minutes until the next arrival (that is, determine $P(T > 0.25)$)?

- A. 0.5 B. 0.3435 C. 0.6065 D. 0.7788 E. none of the above

9. Multiple Choice Question

An insurance company divides its customers into three classes: with low risk, medium risk and high risk. For each group, the probability that a person has at least one accident within a year is, respectively, 0.05, 0.15 and 0.25. It is estimated that 50% of the population is in low risk group, 35% in medium risk group, and 15% in the high risk group. One person is selected at random. What is the probability that this person will have at least one accident within a year?

- A. 0.215 B. 0.115 C. 0.45 D. 0.095 E. none of the preceding

10. Multiple Choice Question

Consider the discrete random variables X and Y with the following joint probability mass function:

x	y	$f_{XY}(x, y)$
-1	0	1/8
0	-1	1/4
0	1	1/4
1	0	1/8
-1	1	1/8
1	-1	1/8

What is $P(X = 1|Y = 0)$? Are X and Y independent?

- A. 0; independent B. 1/2; independent C. 1/2; dependent D. 1/8; dependent
E. none of the preceding

11. Multiple Choice Question

In a nickel-cadmium battery NiCd, a fully charged cell consists of nickelic hydroxide. Nickel is an element which has multiple oxidation states. Let X be the charge of nickel, which has the following probability mass function:

x	$f_X(x)$
0	0.18
1	0.34
2	0.33
4	0.15

Determine the mean charge of nickel.

- A. 2.0 B. 1.5 C. 1.6 D. 1.45 E. none of the preceding

12. Multiple Choice Question

We measured the compressive strength for $n = 16$ specimens of concrete. Using the mean and standard deviation and assuming a normal population, we computed the following confidence interval $[2271.7688, 2308.2312]$. This interval is a confidence interval for the mean compressive strength at a level of confidence of 90%. We are told the the sample mean is $\bar{x} = 2290$. What is the value of the sample standard deviation s ?

A. 44.33 B. 41.6 C. 83.2 D. 54.38 E. none of the preceding

13. Multiple Choice Question

Suppose that the number of bad cheques received by a bank in one day is a Poisson random variable with mean $\lambda = 3$. Determine the probability that the bank will receive 4 bad cheques in 2 days.

A. 0.134 B. 0.058 C. 0.316 D. 0.205 E. none of the above

14. Multiple Choice Question

The lifetime of a voltage regulator for a car follows an exponential distrubution with a mean of 6 years. The number of failures follows a Poisson distribution with a mean of one failure every 6 years. A person buys a 6-year-old car and counts on owning it for another 6 years. If the regulator suffers a failure 3 years after the purchase, compute the average number of years until the next failure.

A. 6 B. 0 C. 3 D. 12 E. none of the preceding

15. Multiple Choice Question

Let X equal the excess weight of a "1000 grams" bottle of soap. Assume that X follows a normal distribution with variance 169g^2 . What sample size is required to have a level of confidence of 95% that the maximum error of the estimate of the mean of the excess weight is less than 1.5g ?

A. 302 B. 287 C. 289 D. 301 E. 288

16. Multiple Choice Question

Consider the following random sample: 5, 34, 12, 10, 4. Calculate the mean and variance of the sample, respectively.

A. 13; 149 B. 13; 119.2 C. 12; 119.2 D. 13; 12.2
E. none of the preceding

17. Multiple Choice Question

The air pressure in a random tire on a certain new car model is normally distributed with a mean of 31lb/in² and a standard deviation of 0.5lb/in². What is the probability that the pressure of a randomly selected tire falls between 30.5 and 31.5 lb/in²?

- A. 0.6827 B. 0.3173 C. 0.5000 D. 0.4245 E. none of the preceding

18. Multiple Choice Question

From a group of 25 members of a Board of Directors, in how many ways one can select a vice-president and a president? (vice-president and president are different persons).

- A. 25 B. 50 C. 600 D. 625 E. none of the preceding

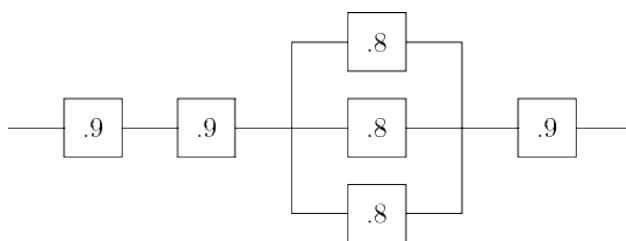
19. Multiple Choice Question

Previous experience has shown that the fracture resistance of a wire used in the manufacture of drapery is normally distributed with $\sigma^2 = 2$. A random sample of 25 specimens have been examined and they yielded a mean resistance of $\bar{x} = 98$ psi. Give a 95% confidence interval for the mean resistance.

- A. [97.216,98.784] B. [97.216,98.554] C. [97.456,98.784] D. [97.446,98.554]
E. none of the preceding

20. Multiple Choice Question

Consider the following system with six components. It operates only if there is a path of functional components from left to the right. The probability that each components functions is as shown. What is the probability that the circuit operates? Assume independence.



- A. 0.723 B. 0.373 C. 0.511 D. 0.633 E. none of the preceding

21. True or False Question

In order to establish a control chart for the mean of a process, 20 samples each of size 4 are collected. We note that $\sum_{i=1}^{20} \bar{x}_i = 4000$ and $\sum_{i=1}^{20} s_i = 500$. The value of the upper control limit of the chart for the mean is approximately equal to 204.5.

True False

22. True or False Question

The percentage of males in 1986 who are 18-19 years old and married was 3.7%. To test whether or not this percentage had increased in 2015, a random sample of 300 males aged 18-19 was taken; 20 of which were married. Using a level of significance of $\alpha = 0.05$, we test the hypothesis H_0 that the proportion that is married is 3.7% against the alternative that it is larger. Based on the data, the observed value of the corresponding test statistic is 2.722 and we reject H_0 .

True False

23. True or False Question

A new treatment has been developed for a certain type of cement. To test the null hypothesis that $\mu = 5000$ against the alternative that $\mu \neq 5000$, where μ is the mean resistance in kg/cm², a sample of 50 samples of cement is examined and we observe $\bar{x} = 4775$ and $s = 130$. At $\alpha = 0.05$, the evidence against the null hypothesis in favour of the alternative hypothesis is significant.

True False

24. True or False Question

Let X and Y be random variables with joint density function

$$f(x, y) = \begin{cases} 6y & 0 < y \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

The marginal density of Y is $f_Y(y) = 3y(1 - y)$, for $0 < y < 1$.

True False

25. Short Answer Question

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned} \sum_{i=1}^n x_i &= 683, \quad \sum_{i=1}^n y_i = 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731. \end{aligned}$$

What is the mean squared error estimate for the variance of the residuals?

26. Short Answer Question

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\sum_{i=1}^n x_i = 683, \quad \sum_{i=1}^n y_i = 813,$$

$$\sum_{i=1}^n x_i^2 = 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731.$$

What is an approximate 95% confidence interval for the mean response at $x_0 = 60$?

27. Short Answer Question

We have $m = 5$ preliminary samples of size $n = 3$ (some numbers have unfortunately been erased by accident by a clumsy co-op student):

i	$x_{i,1}$	$x_{i,2}$	$x_{i,3}$	$\bar{x}_i$	r_i	s_i
1	27.1	29.4		27.9		1.3
2	30.6	32.5	32.4	31.83	1.9	1.07
3	25.7	35.5	30	30.4		4.91
4	31.1	23.2	25	26.43	7.9	
5	24.1	34.2	27.4	28.57	10.1	5.15
total:				145.13	32	16.57

What is the control chart (give the interval) for $\bar{X}$ from $\bar{S}$?

28. Short Answer Question

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\sum_{i=1}^n x_i = 683, \quad \sum_{i=1}^n y_i = 813,$$

$$\sum_{i=1}^n x_i^2 = 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731.$$

What is the line of best fit for this data?

29. Short Answer Question

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\sum_{i=1}^n x_i = 683, \quad \sum_{i=1}^n y_i = 813,$$

$$\sum_{i=1}^n x_i^2 = 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731.$$

What is an approximate 99% confidence interval for the intercept of the line of best fit?

30. Short Answer Question

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\sum_{i=1}^n x_i = 683, \quad \sum_{i=1}^n y_i = 813,$$

$$\sum_{i=1}^n x_i^2 = 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731.$$

What is an approximate 99% confidence interval for the slope of the line of best fit?

31. Short Answer Question

We have $m = 5$ preliminary samples of size $n = 3$ (some numbers have unfortunately been erased by accident by a clumsy co-op student):

i	$x_{i,1}$	$x_{i,2}$	$x_{i,3}$	$\bar{x}_i$	r_i	s_i
1	27.1	29.4		27.9		1.3
2	30.6	32.5	32.4	31.83	1.9	1.07
3	25.7	35.5	30	30.4		4.91
4	31.1	23.2	25	26.43	7.9	
5	24.1	34.2	27.4	28.57	10.1	5.15
total:				145.13	32	16.57

What is the control chart (give the interval) for $\bar{X}$ from $\bar{R}$?

32. Short Answer Question

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\sum_{i=1}^n x_i = 683, \quad \sum_{i=1}^n y_i = 813,$$

$$\sum_{i=1}^n x_i^2 = 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731.$$

What is an approximate 99% prediction interval for the response y_0 at $x_0 = 60$?

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>									
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
8	0	0.4305	0.1678	0.1001	0.0576	0.0168	0.0039	0.0007	0.0001	0.0000	
	1	0.8131	0.5033	0.3671	0.2553	0.1064	0.0352	0.0085	0.0013	0.0001	
	2	0.9619	0.7969	0.6785	0.5518	0.3154	0.1445	0.0498	0.0113	0.0012	0.0000
	3	0.9950	0.9437	0.8862	0.8059	0.5941	0.3633	0.1737	0.0580	0.0104	0.0004
	4	0.9996	0.9896	0.9727	0.9420	0.8263	0.6367	0.4059	0.1941	0.0563	0.0050
	5	1.0000	0.9988	0.9958	0.9887	0.9502	0.8555	0.6846	0.4482	0.2031	0.0381
	6		0.9999	0.9996	0.9987	0.9915	0.9648	0.8936	0.7447	0.4967	0.1869
	7		1.0000	1.0000	0.9999	0.9993	0.9961	0.9832	0.9424	0.8322	0.5695
	8				1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	0	0.3874	0.1342	0.0751	0.0404	0.0101	0.0020	0.0003	0.0000		
	1	0.7748	0.4362	0.3003	0.1960	0.0705	0.0195	0.0038	0.0004	0.0000	
	2	0.9470	0.7382	0.6007	0.4628	0.2318	0.0898	0.0250	0.0043	0.0003	0.0000
	3	0.9917	0.9144	0.8343	0.7297	0.4826	0.2539	0.0994	0.0253	0.0031	0.0001
	4	0.9991	0.9804	0.9511	0.9012	0.7334	0.5000	0.2666	0.0988	0.0196	0.0009
	5	0.9999	0.9969	0.9900	0.9747	0.9006	0.7461	0.5174	0.2703	0.0856	0.0083
	6	1.0000	0.9997	0.9987	0.9957	0.9750	0.9102	0.7682	0.5372	0.2618	0.0530
	7		1.0000	0.9999	0.9996	0.9962	0.9805	0.9295	0.8040	0.5638	0.2252
	8			1.0000	1.0000	0.9997	0.9980	0.9899	0.9596	0.8658	0.6126
	9					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0	0.3487	0.1074	0.0563	0.0282	0.0060	0.0010	0.0001	0.0000		
	1	0.7361	0.3758	0.2440	0.1493	0.0464	0.0107	0.0017	0.0001	0.0000	
	2	0.9298	0.6778	0.5256	0.3828	0.1673	0.0547	0.0123	0.0016	0.0001	
	3	0.9872	0.8791	0.7759	0.6496	0.3823	0.1719	0.0548	0.0106	0.0009	0.0000
	4	0.9984	0.9672	0.9219	0.8497	0.6331	0.3770	0.1662	0.0473	0.0064	0.0001
	5	0.9999	0.9936	0.9803	0.9527	0.8338	0.6230	0.3669	0.1503	0.0328	0.0016
	6	1.0000	0.9991	0.9965	0.9894	0.9452	0.8281	0.6177	0.3504	0.1209	0.0128
	7		0.9999	0.9996	0.9984	0.9877	0.9453	0.8327	0.6172	0.3222	0.0702
	8		1.0000	1.0000	0.9999	0.9983	0.9893	0.9536	0.8507	0.6242	0.2639
	9				1.0000	0.9999	0.9990	0.9940	0.9718	0.8926	0.6513
	10					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0	0.3138	0.0859	0.0422	0.0198	0.0036	0.0005	0.0000			
	1	0.6974	0.3221	0.1971	0.1130	0.0302	0.0059	0.0007	0.0000		
	2	0.9104	0.6174	0.4552	0.3127	0.1189	0.0327	0.0059	0.0006	0.0000	
	3	0.9815	0.8389	0.7133	0.5696	0.2963	0.1133	0.0293	0.0043	0.0002	
	4	0.9972	0.9496	0.8854	0.7897	0.5328	0.2744	0.0994	0.0216	0.0020	0.0000
	5	0.9997	0.9883	0.9657	0.9218	0.7535	0.5000	0.2465	0.0782	0.0117	0.0003
	6	1.0000	0.9980	0.9924	0.9784	0.9006	0.7256	0.4672	0.2103	0.0504	0.0028
	7		0.9998	0.9988	0.9957	0.9707	0.8867	0.7037	0.4304	0.1611	0.0185
	8		1.0000	0.9999	0.9994	0.9941	0.9673	0.8811	0.6873	0.3826	0.0896
	9			1.0000	1.0000	0.9993	0.9941	0.9698	0.8870	0.6779	0.3026
	10					1.0000	0.9995	0.9964	0.9802	0.9141	0.6862
	11						1.0000	1.0000	1.0000	1.0000	1.0000

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>									
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
12	0	0.2824	0.0687	0.0317	0.0138	0.0022	0.0002	0.0000			
	1	0.6590	0.2749	0.1584	0.0850	0.0196	0.0032	0.0003	0.0000		
	2	0.8891	0.5583	0.3907	0.2528	0.0834	0.0193	0.0028	0.0002	0.0000	
	3	0.9744	0.7946	0.6488	0.4925	0.2253	0.0730	0.0153	0.0017	0.0001	
	4	0.9957	0.9274	0.8424	0.7237	0.4382	0.1938	0.0573	0.0095	0.0006	0.0000
	5	0.9995	0.9806	0.9456	0.8822	0.6652	0.3872	0.1582	0.0386	0.0039	0.0001
	6	0.9999	0.9961	0.9857	0.9614	0.8418	0.6128	0.3348	0.1178	0.0194	0.0005
	7	1.0000	0.9994	0.9972	0.9905	0.9427	0.8062	0.5618	0.2763	0.0726	0.0043
	8		0.9999	0.9996	0.9983	0.9847	0.9270	0.7747	0.5075	0.2054	0.0256
	9		1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.4417	0.1109
	10				1.0000	0.9997	0.9968	0.9804	0.9150	0.7251	0.3410
	11					1.0000	0.9998	0.9978	0.9862	0.9313	0.7176
	12						1.0000	1.0000	1.0000	1.0000	1.0000
13	0	0.2542	0.0550	0.0238	0.0097	0.0013	0.0001	0.0000			
	1	0.6213	0.2336	0.1267	0.0637	0.0126	0.0017	0.0001	0.0000		
	2	0.8661	0.5017	0.3326	0.2025	0.0579	0.0112	0.0013	0.0001		
	3	0.9658	0.7473	0.5843	0.4206	0.1686	0.0461	0.0078	0.0007	0.0000	
	4	0.9935	0.9009	0.7940	0.6543	0.3530	0.1334	0.0321	0.0040	0.0002	
	5	0.9991	0.9700	0.9198	0.8346	0.5744	0.2905	0.0977	0.0182	0.0012	0.0000
	6	0.9999	0.9930	0.9757	0.9376	0.7712	0.5000	0.2288	0.0624	0.0070	0.0001
	7	1.0000	0.9988	0.9944	0.9818	0.9023	0.7095	0.4256	0.1654	0.0300	0.0009
	8		0.9998	0.9990	0.9960	0.9679	0.8666	0.6470	0.3457	0.0991	0.0065
	9		1.0000	0.9999	0.9993	0.9922	0.9539	0.8314	0.5794	0.2527	0.0342
	10			1.0000	0.9999	0.9987	0.9888	0.9421	0.7975	0.4983	0.1339
	11				1.0000	0.9999	0.9983	0.9874	0.9363	0.7664	0.3787
	12					1.0000	0.9999	0.9987	0.9903	0.9450	0.7458
	13						1.0000	1.0000	1.0000	1.0000	1.0000
14	0	0.2288	0.0440	0.0178	0.0068	0.0008	0.0001	0.0000			
	1	0.5846	0.1979	0.1010	0.0475	0.0081	0.0009	0.0001			
	2	0.8416	0.4481	0.2811	0.1608	0.0398	0.0065	0.0006	0.0000		
	3	0.9559	0.6982	0.5213	0.3552	0.1243	0.0287	0.0039	0.0002		
	4	0.9908	0.8702	0.7415	0.5842	0.2793	0.0898	0.0175	0.0017	0.0000	
	5	0.9985	0.9561	0.8883	0.7805	0.4859	0.2120	0.0583	0.0083	0.0004	
	6	0.9998	0.9884	0.9617	0.9067	0.6925	0.3953	0.1501	0.0315	0.0024	0.0000
	7	1.0000	0.9976	0.9897	0.9685	0.8499	0.6047	0.3075	0.0933	0.0116	0.0002
	8		0.9996	0.9978	0.9917	0.9417	0.7880	0.5141	0.2195	0.0439	0.0015
	9		1.0000	0.9997	0.9983	0.9825	0.9102	0.7207	0.4158	0.1298	0.0092
	10			1.0000	0.9998	0.9961	0.9713	0.8757	0.6448	0.3018	0.0441
	11				1.0000	0.9994	0.9935	0.9602	0.8392	0.5519	0.1584
	12					0.9999	0.9991	0.9919	0.9525	0.8021	0.4154
	13					1.0000	0.9999	0.9992	0.9932	0.9560	0.7712
	14						1.0000	1.0000	1.0000	1.0000	1.0000

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>									
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
15	0	0.2059	0.0352	0.0134	0.0047	0.0005	0.0000				
	1	0.5490	0.1671	0.0802	0.0353	0.0052	0.0005	0.0000			
	2	0.8159	0.3980	0.2361	0.1268	0.0271	0.0037	0.0003	0.0000		
	3	0.9444	0.6482	0.4613	0.2969	0.0905	0.0176	0.0019	0.0001		
	4	0.9873	0.8358	0.6865	0.5155	0.2173	0.0592	0.0093	0.0007	0.0000	
	5	0.9978	0.9389	0.8516	0.7216	0.4032	0.1509	0.0338	0.0037	0.0001	
	6	0.9997	0.9819	0.9434	0.8689	0.6098	0.3036	0.0950	0.0152	0.0008	
	7	1.0000	0.9958	0.9827	0.9500	0.7869	0.5000	0.2131	0.0500	0.0042	0.0000
	8		0.9992	0.9958	0.9848	0.9050	0.6964	0.3902	0.1311	0.0181	0.0003
	9		0.9999	0.9992	0.9963	0.9662	0.8491	0.5968	0.2784	0.0611	0.0022
	10		1.0000	0.9999	0.9993	0.9907	0.9408	0.7827	0.4845	0.1642	0.0127
	11			1.0000	0.9999	0.9981	0.9824	0.9095	0.7031	0.3518	0.0556
	12				1.0000	0.9997	0.9963	0.9729	0.8732	0.6020	0.1841
	13					1.0000	0.9995	0.9948	0.9647	0.8329	0.4510
	14						1.0000	0.9995	0.9953	0.9648	0.7941
	15							1.0000	1.0000	1.0000	1.0000
16	0	0.1853	0.0281	0.0100	0.0033	0.0003	0.0000				
	1	0.5147	0.1407	0.0635	0.0261	0.0033	0.0003	0.0000			
	2	0.7892	0.3518	0.1971	0.0994	0.0183	0.0021	0.0001			
	3	0.9316	0.5981	0.4050	0.2459	0.0651	0.0106	0.0009	0.0000		
	4	0.9830	0.7982	0.6302	0.4499	0.1666	0.0384	0.0049	0.0003		
	5	0.9967	0.9183	0.8103	0.6598	0.3288	0.1051	0.0191	0.0016	0.0000	
	6	0.9995	0.9733	0.9204	0.8247	0.5272	0.2272	0.0583	0.0071	0.0002	
	7	0.9999	0.9930	0.9729	0.9256	0.7161	0.4018	0.1423	0.0257	0.0015	0.0000
	8	1.0000	0.9985	0.9925	0.9743	0.8577	0.5982	0.2839	0.0744	0.0070	0.0001
	9		0.9998	0.9984	0.9929	0.9417	0.7728	0.4728	0.1753	0.0267	0.0005
	10		1.0000	0.9997	0.9984	0.9809	0.8949	0.6712	0.3402	0.0817	0.0033
	11			1.0000	0.9997	0.9951	0.9616	0.8334	0.5501	0.2018	0.0170
	12				1.0000	0.9991	0.9894	0.9349	0.7541	0.4019	0.0684
	13					0.9999	0.9979	0.9817	0.9006	0.6482	0.2108
	14					1.0000	0.9997	0.9967	0.9739	0.8593	0.4853
	15						1.0000	0.9997	0.9967	0.9719	0.8147
	16							1.0000	1.0000	1.0000	1.0000

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>									
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
17	0	0.1668	0.0225	0.0075	0.0023	0.0002	0.0000				
	1	0.4818	0.1182	0.0501	0.0193	0.0021	0.0001	0.0000			
	2	0.7618	0.3096	0.1637	0.0774	0.0123	0.0012	0.0001			
	3	0.9174	0.5489	0.3530	0.2019	0.0464	0.0064	0.0005	0.0000		
	4	0.9779	0.7582	0.5739	0.3887	0.1260	0.0245	0.0025	0.0001		
	5	0.9953	0.8943	0.7653	0.5968	0.2639	0.0717	0.0106	0.0007	0.0000	
	6	0.9992	0.9623	0.8929	0.7752	0.4478	0.1662	0.0348	0.0032	0.0001	
	7	0.9999	0.9891	0.9598	0.8954	0.6405	0.3145	0.0919	0.0127	0.0005	
	8	1.0000	0.9974	0.9876	0.9597	0.8011	0.5000	0.1989	0.0403	0.0026	0.0000
	9		0.9995	0.9969	0.9873	0.9081	0.6855	0.3595	0.1046	0.0109	0.0001
	10		0.9999	0.9994	0.9968	0.9652	0.8338	0.5522	0.2248	0.0377	0.0008
	11		1.0000	0.9999	0.9993	0.9894	0.9283	0.7361	0.4032	0.1057	0.0047
	12			1.0000	0.9999	0.9975	0.9755	0.8740	0.6113	0.2418	0.0221
	13				1.0000	0.9995	0.9936	0.9536	0.7981	0.4511	0.0826
	14					0.9999	0.9988	0.9877	0.9226	0.6904	0.2382
	15					1.0000	0.9999	0.9979	0.9807	0.8818	0.5182
	16						1.0000	0.9998	0.9977	0.9775	0.8332
	17							1.0000	1.0000	1.0000	1.0000
18	0	0.1501	0.0180	0.0056	0.0016	0.0001	0.0000				
	1	0.4503	0.0991	0.0395	0.0142	0.0013	0.0001				
	2	0.7338	0.2713	0.1353	0.0600	0.0082	0.0007	0.0000			
	3	0.9018	0.5010	0.3057	0.1646	0.0328	0.0038	0.0002			
	4	0.9718	0.7164	0.5187	0.3327	0.0942	0.0154	0.0013	0.0000		
	5	0.9936	0.8671	0.7175	0.5344	0.2088	0.0481	0.0058	0.0003		
	6	0.9988	0.9487	0.8610	0.7217	0.3743	0.1189	0.0203	0.0014	0.0000	
	7	0.9998	0.9837	0.9431	0.8593	0.5634	0.2403	0.0576	0.0061	0.0002	
	8	1.0000	0.9957	0.9807	0.9404	0.7368	0.4073	0.1347	0.0210	0.0009	
	9		0.9991	0.9946	0.9790	0.8653	0.5927	0.2632	0.0596	0.0043	0.0000
	10		0.9998	0.9988	0.9939	0.9424	0.7597	0.4366	0.1407	0.0163	0.0002
	11		1.0000	0.9998	0.9986	0.9797	0.8811	0.6257	0.2783	0.0513	0.0012
	12			1.0000	0.9997	0.9942	0.9519	0.7912	0.4656	0.1329	0.0064
	13				1.0000	0.9987	0.9846	0.9058	0.6673	0.2836	0.0282
	14					0.9998	0.9962	0.9672	0.8354	0.4990	0.0982
	15					1.0000	0.9993	0.9918	0.9400	0.7287	0.2662
	16						0.9999	0.9987	0.9858	0.9009	0.5497
	17						1.0000	0.9999	0.9984	0.9820	0.8499
	18							1.0000	1.0000	1.0000	1.0000

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>									
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
19	0	0.1351	0.0144	0.0042	0.0011	0.0001					
	1	0.4203	0.0829	0.0310	0.0104	0.0008	0.0000				
	2	0.7054	0.2369	0.1113	0.0462	0.0055	0.0004	0.0000			
	3	0.8850	0.4551	0.2631	0.1332	0.0230	0.0022	0.0001			
	4	0.9648	0.6733	0.4654	0.2822	0.0696	0.0096	0.0006	0.0000		
	5	0.9914	0.8369	0.6678	0.4739	0.1629	0.0318	0.0031	0.0001		
	6	0.9983	0.9324	0.8251	0.6655	0.3081	0.0835	0.0116	0.0006		
	7	0.9997	0.9767	0.9225	0.8180	0.4878	0.1796	0.0352	0.0028	0.0000	
	8	1.0000	0.9933	0.9713	0.9161	0.6675	0.3238	0.0885	0.0105	0.0003	
	9		0.9984	0.9911	0.9674	0.8139	0.5000	0.1861	0.0326	0.0016	
	10		0.9997	0.9977	0.9895	0.9115	0.6762	0.3325	0.0839	0.0067	0.0000
	11		1.0000	0.9995	0.9972	0.9648	0.8204	0.5122	0.1820	0.0233	0.0003
	12			0.9999	0.9994	0.9884	0.9165	0.6919	0.3345	0.0676	0.0017
	13			1.0000	0.9999	0.9969	0.9682	0.8371	0.5261	0.1631	0.0086
	14				1.0000	0.9994	0.9904	0.9304	0.7178	0.3267	0.0352
	15					0.9999	0.9978	0.9770	0.8668	0.5449	0.1150
	16					1.0000	0.9996	0.9945	0.9538	0.7631	0.2946
	17						1.0000	0.9992	0.9896	0.9171	0.5797
	18							0.9999	0.9989	0.9856	0.8649
	19							1.0000	1.0000	1.0000	1.0000
20	0	0.1216	0.0115	0.0032	0.0008	0.0000					
	1	0.3917	0.0692	0.0243	0.0076	0.0005	0.0000				
	2	0.6769	0.2061	0.0913	0.0355	0.0036	0.0002				
	3	0.8670	0.4114	0.2252	0.1071	0.0160	0.0013	0.0000			
	4	0.9568	0.6296	0.4148	0.2375	0.0510	0.0059	0.0003			
	5	0.9887	0.8042	0.6172	0.4164	0.1256	0.0207	0.0016	0.0000		
	6	0.9976	0.9133	0.7858	0.6080	0.2500	0.0577	0.0065	0.0003		
	7	0.9996	0.9679	0.8982	0.7723	0.4159	0.1316	0.0210	0.0013	0.0000	
	8	0.9999	0.9900	0.9591	0.8867	0.5956	0.2517	0.0565	0.0051	0.0001	
	9	1.0000	0.9974	0.9861	0.9520	0.7553	0.4119	0.1275	0.0171	0.0006	
	10		0.9994	0.9961	0.9829	0.8725	0.5881	0.2447	0.0480	0.0026	0.0000
	11		0.9999	0.9991	0.9949	0.9435	0.7483	0.4044	0.1133	0.0100	0.0001
	12		1.0000	0.9998	0.9987	0.9790	0.8684	0.5841	0.2277	0.0321	0.0004
	13			1.0000	0.9997	0.9935	0.9423	0.7500	0.3920	0.0867	0.0024
	14				1.0000	0.9984	0.9793	0.8744	0.5836	0.1958	0.0113
	15					0.9997	0.9941	0.9490	0.7625	0.3704	0.0432
	16					1.0000	0.9987	0.9840	0.8929	0.5886	0.1330
	17						0.9998	0.9964	0.9645	0.7939	0.3231
	18						1.0000	0.9995	0.9924	0.9308	0.6083
	19							1.0000	0.9992	0.9885	0.8784
	20								1.0000	1.0000	1.0000

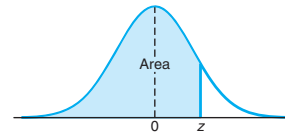
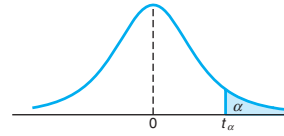


Table A.3 Areas under the Normal Curve

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

**Table A.4** Critical Values of the t -Distribution

v	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Control Chart Constants

n	A_3	B_3	B_4	A_2	D_3	D_4
4	1.63	0	2.27	0.73	0	2.28
5	1.43	0	2.09	0.58	0	2.11
6	1.29	0.03	1.97	0.48	0	2.00
8	1.10	0.185	1.815	0.37	0.14	1.86
10	0.98	0.28	1.72	0.31	0.22	1.78
20	0.68	0.51	1.49	0.18	0.41	1.59

Sample Size = n	A_2	A_3
2	1.880	2.659
3	1.023	1.954
4	0.729	1.628
5	0.577	1.427
6	0.483	1.287
7	0.419	1.182
8	0.373	1.099
9	0.337	1.032
10	0.308	0.975
11	0.285	0.927
12	0.266	0.886
13	0.249	0.850
14	0.235	0.817
15	0.223	0.789
16	0.212	0.763
17	0.203	0.739
18	0.194	0.718
19	0.187	0.698
20	0.180	0.680
21	0.173	0.663
22	0.167	0.647
23	0.162	0.633
24	0.157	0.619
25	0.153	0.606