

MAT 2377

Winter 2020

Final Exam

April 20, 2020

Time Limit: 3 Hours

Final Examination

Version: 27866

Start Time: 2PM EST

This exam contains 37 pages (including this cover page and tables) and 110 questions.

Read carefully: if you submit answers for this online examination, you promise and certify that

- all work done during the assigned exam period will be done entirely by yourself, with no help from others;
- you will not communicate with anybody except the professor during the exam, for exam- and course-related questions;
- you will not consult any people, sources, or writings other than the course textbook, your self-made review sheets, your course notes, your prepared answers, the course documents made available on Brightspace, and the exam itself;
- you will not provide information related to the exam's contents to other people until 24 hours after the exam is over.

Obviously, we do not have a way to monitor your adherence to these instructions (nor do we really wish to look for one); we will be asking, instead, for your word of honour that you will follow the directives and work on this examination alone.

Multiple choice questions and true or false question are worth 1 mark each; short answer questions, 2 marks. There are no part marks. For each question, record your answer in the corresponding space on the answer sheet that has been provided separately (save your file regularly). For multiple choice questions, select the closest answer among the options, when appropriate; for short answer questions, provide an answer to at least 2 decimals.

IMPORTANT: every student has been assigned a different exam (based on the last 5 digits of your student number). You need to answer the questions of the final exam that has been specifically assigned to you.

When you have answered all the questions and you are ready to upload the answers to Brightspace, save your answers to the file `Answer_Sheet_27866.pdf`. We suggest that you print and scan or take a screenshot (with time stamp) of your completed answer sheet before uploading to Brightspace (see the appropriate location in the "Assignment" section where you uploaded your 3rd assignment), and that you keep a copy of these documents (with time stamp). Should Brightspace's upload functionality does not work when you are ready to submit, wait a little bit and try again; as a last recourse, please email your answer sheet with saved answers to `pboily@uottawa.ca` if you are registered in section A or `iyadegar@uottawa.ca` if you are registered in section B.

No worries. You've got this.

1. [1] **Multiple Choice Question**

An insurance company divides its customers into three classes: with low risk, medium risk and high risk. For each group, the probability that a person has at least one accident within a year is, respectively, 0.05, 0.15 and 0.25. It is estimated that 50% of the population is in low risk group, 35% in medium risk group, and 15% in the high risk group. One person is selected at random. What is the probability that this person will have at least one accident within a year?

A. 0.215 B. 0.115 C. 0.45 D. 0.095 E. none of the preceding

2. [1] **Multiple Choice Question**

From a group of 25 members of a Board of Directors, in how many ways one can select a vice-president and a president? (vice-president and president are different persons).

A. 25 B. 50 C. 600 D. 625 E. none of the preceding

3. [1] **Multiple Choice Question**

In how many ways can we select four out of 12 companies renting construction equipment to be ranked as first, second, third and fourth?

A. 11880 B. 20736 C. 495 D. 12 E. none of the preceding

4. [1] **Multiple Choice Question**

In a pile of 20 tires, three of them are defective. Four tires are selected randomly for inspection. What is the probability that exactly one is defective?

A. 0.15 B. 0.42 C. 0.1053 D. 0.4912 E. none of the preceding

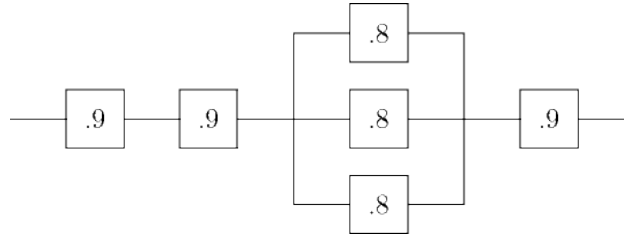
5. [1] **Multiple Choice Question**

A company has four technicians. The technicians T1, T2, T3 and T4 repair 20%, 15%, 50% and 15% of failures, respectively. Suppose that these technicians do not complete their repairs in 3%, 1%, 2% and 1% cases, respectively. If a repair has not been completed, what is the probability that this job was conducted by technician T1?

A. 0.019 B. 0.05 C. 0.316 D. 0.20 E. none of the preceding

6. [1] **Multiple Choice Question**

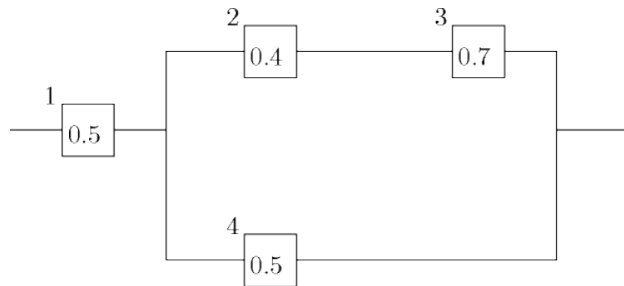
Consider the following system with six components. It operates only if there is a path of functional components from left to the right. The probability that each components functions is as shown. What is the probability that the circuit operates? Assume independence.



A. 0.723 B. 0.373 C. 0.511 D. 0.633 E. none of the preceding

7. [1] **Multiple Choice Question**

Consider the following system with four components. It operates only if there is a path of functional components from left to the right. The probability that each component functions is as shown. What is the probability that the circuit operates? Assume independence.



A. 0.32 B. 0.16 C. 0.035 D. 0.68 E. none of the preceding

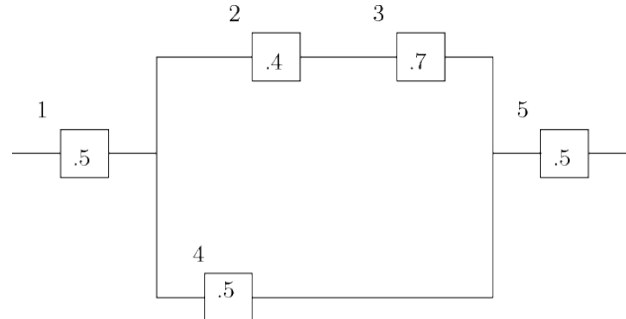
8. [1] **Multiple Choice Question**

A group of 20 workers was surveyed to obtain their opinion about new security measures. If 12 workers are in favour of the new rules and 8 oppose them, what is the probability that two randomly selected workers (from the group of twenty) oppose the new security measures?

A. $\frac{2}{44}$ B. $\frac{14}{95}$ C. $\frac{1}{6}$ D. $\frac{12}{45}$ E. none of the preceding

9. [1] **Multiple Choice Question**

Consider the following system with five components. It operates only if there is a path of functional components from left to the right. The probability that each device functions is as shown. What is the probability that the circuit operates? Assume independence.



- A. 0.84 B. 0.16 C. 0.035 D. 0.50 E. none of the preceding

10. [1] **Multiple Choice Question**

In a study on opposite-sex couples and work, 1000 couples have both the male and the female working. Each person was asked whether his or her salary exceeded \$30,000. The following information was obtained.

	$M \leq \$30K$	$M > \$30K$
$F \leq \$30K$	430	410
$F > \$30K$	60	100

What is the probability that a female earns more than \$30K given that the male earns less than \$30K?

- A. 0.8059 B. 0.1961 C. 0.1224 D. 0.5700 E. none of the preceding

11. [1] **Multiple Choice Question**

Consider a box of 50 fuses which contains 8 faulty fuses. Five fuses are selected at random. What is the probability that none of them is faulty?

- A. 0.4015 B. 0.84 C. 0.3725 D. 0.4275 E. none of the preceding

12. [1] **Multiple Choice Question**

Fifteen individuals volunteered to take part in an experiment. We have 5 treatments. We want to partition the 15 subjects into 5 treatment groups of equal size. In how many different ways can we accomplish this task?

- A. 1024 B. 325 C. 15 D. 168,168,000 E. none of the preceding

13. [1] **Multiple Choice Question**

If A and B are disjoint events, $P(A) = 0.5$, and the probability that at least one of the two events occurs is 0.6, calculate $P(B)$.

- A. 0.6 B. 0.5 C. 0.1 D. 0.9 E. 0.4

14. [1] **Multiple Choice Question**

Studies show that 12% of all patients treated in the ER are admitted to stay overnight. Of the patients treated, 1% have an adverse drug reaction and 12.4% are admitted overnight or have an adverse drug reaction. We select a patient at random. What is the probability that the patient will be admitted to stay overnight and have a drug reaction?

- A. 0.006 B. 0.5 C. 0.875 D. 0.004 E. 0.114

15. [1] **Multiple Choice Question**

Studies show that 12% of all patients treated in the ER are admitted to stay overnight. Of the patients treated, 1% have an adverse drug reaction and 12.4% are admitted overnight or have an adverse drug reaction. We select a patient at random. What is the probability that the patient will be admitted but have no adverse drug reaction?

- A. 0.006 B. 0.5 C. 0.875 D. 0.004 E. 0.114

16. [1] **Multiple Choice Question**

Studies show that 12% of all patients treated in the ER are admitted to stay overnight. Of the patients treated, 1% have an adverse drug reaction and 12.4% are admitted overnight or have an adverse drug reaction. We select a patient at random. What is the probability that the patient will have an adverse drug reaction but won't be admitted overnight?

- A. 0.006 B. 0.5 C. 0.875 D. 0.004 E. 0.114

17. [1] **Multiple Choice Question**

Consider events A and B such that $P(A) = 0.7$, $P(B) = 0.2$ and $P(A \cap B) = 0.3$. Compute the probability that A will occur, given that B does not occur.

- A. 0.4 B. 0.1 C. -0.1 D. 0.5 E. none of the preceding

18. [1] **Multiple Choice Question**

The data collected during an exhibition show that 1% of all donors are HIV positive and 2% were positive for herpes. If 1.5% of all donors were positive for only one or the other (but not both) of these conditions, what is the probability that a randomly selected donor has none of these conditions?

- A. 0.9775 B. 0.9925 C. 0.9850 D. 0.9700 E. 0.9950

19. [1] **Multiple Choice Question**

It was reported that 60% of a particular type of services are financed by private funds, 70% are funded by employers and employees, and 40% are financed by private funds and by the employers and employees. What is the probability that an individual that is randomly accessing such a service will choose a service that is funded by employers and employees but not by private funds?

A. 0.30 B. 0.20 C. 0.10 D. 0.50 E. none of the preceding

20. [1] **Multiple Choice Question**

On average, 1 in 100 people have a particular mutated gene. 600 people are examined. Approximate the probability that 5 or more have the gene.

A. 0.10 B. 0.90 C. 0.73 D. 0.26 E. none of the preceding

21. [1] **Multiple Choice Question**

Suppose X is a random variable with probability mass function

$$f(x) = \begin{cases} k & \text{if } x = 0 \\ k & \text{if } x = 1 \\ 14/25 & \text{if } x = 3 \end{cases}$$

Find k .

A. 11/50 B. 11/25 C. 22/25 D. 22/50 E. none of the preceding

22. [1] **Multiple Choice Question**

Let X be a continuous random variable with density $f(x) = e^{-3x}$, $x > a$. Find a .

A. $(\ln 3)/3$ B. $-(\ln 3)/3$ C. 0 D. 3 E. none of the preceding

23. [1] **Multiple Choice Question**

Suppose X is a random variable with probability mass function

$$f(x) = \begin{cases} k & \text{if } x = 0 \\ k & \text{if } x = 1 \\ 3k & \text{if } x = 2 \\ 14/25 & \text{if } x = 3 \end{cases}$$

Find k .

A. 11/500 B. 11/250 C. 11/125 D. 11/120 E. none of the preceding

24. [1] **Multiple Choice Question**

The lifetime of a voltage regulator for a car follows an exponential distribution with a mean of 6 years. The number of failures follows a Poisson distribution with a mean of one failure every 6 years. A person buys a 6-year-old car and counts on owning it for another 6 years. If the regulator suffers a failure 3 years after the purchase, compute the average number of years until the next failure.

A. 6 B. 0 C. 3 D. 12 E. none of the preceding

25. [1] **Multiple Choice Question**

The air pressure in a random tire on a certain new car model is normally distributed with a mean of 31lb/in² and a standard deviation of 0.5lb/in². What is the probability that the pressure of a randomly selected tire falls between 30.5 and 31.5 lb/in²?

A. 0.6827 B. 0.3173 C. 0.5000 D. 0.4245 E. none of the preceding

26. [1] **Multiple Choice Question**

Trucks arrive at a loading/unloading station according to a Poisson process with a rate of 2 trucks per hour. Determine the probability that at least 3 trucks will arrive at the station in the next 30 minutes.

A. 0.86 B. 0.59 C. 0.13 D. 0.81 E. 0.08

27. [1] **Multiple Choice Question**

Trucks arrive at a loading/unloading station according to a Poisson process with a rate of 2 trucks per hour. A truck just arrived at the station. A station employee would like to know if he has enough time to take a break. Let the random variable T represent the waiting time (in hours) until the next truck arrival. What is the probability that the employee has 15 minutes until the next arrival (that is, determine $P(T > 0.25)$)?

A. 0.5 B. 0.3435 C. 0.6065 D. 0.7788 E. none of the above

28. [1] **Multiple Choice Question**

In a nickel-cadmium battery NiCd, a fully charged cell consists of nickelic hydroxide. Nickel is an element which has multiple oxidation states. Let X be the charge of nickel, which has the following probability mass function:

x	$f_X(x)$
0	0.18
1	0.34
2	0.33
4	0.15

Determine the mean charge of nickel.

A. 2.0 B. 1.5 C. 1.6 D. 1.45 E. none of the preceding

29. [1] **Multiple Choice Question**

Let X be the number of failures for a certain machine during a month. Its cumulative distribution function is

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.17 & \text{if } 0 \leq x < 1 \\ 0.40 & \text{if } 1 \leq x < 2 \\ 0.59 & \text{if } 2 \leq x < 3 \\ 0.72 & \text{if } 3 \leq x < 4 \\ 0.80 & \text{if } 4 \leq x < 5 \\ 1 & \text{if } x \geq 5 \end{cases}$$

Compute the probability that there will be more than 3 failures during a month.

A. 0.28 B. 0.72 C. 0.20 D. 0.80 E. 0.41

30. [1] **Multiple Choice Question**

Let X be the number of failures for a certain machine during a month. Its cumulative distribution function is

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ 0.17 & \text{if } 0 \leq x < 1 \\ 0.40 & \text{if } 1 \leq x < 2 \\ 0.59 & \text{if } 2 \leq x < 3 \\ 0.72 & \text{if } 3 \leq x < 4 \\ 0.80 & \text{if } 4 \leq x < 5 \\ 1 & \text{if } x \geq 5 \end{cases}$$

What is the expected number of failures for a month?

A. 2.50 B. 3.00 C. 2.32 D. 11.94 E. none of the above

31. [1] **Multiple Choice Question**

Suppose that 10% of the chips produced by a computer hardware factory are defective. If you order 5 chips, what is the probability of receiving at most one defective chip?

A. 0.9000 B. 0.6561 C. 0.9185 D. 0.3280 E. none of the above

32. [1] **Multiple Choice Question**

The lifetime (in 1,000 km) for a certain brand of tire is modeled as a continuous random variable with the following cumulative distribution function: $F(x) = 1 - e^{-(x-10)/50}$, $x > 10$. Determine a lifetime x (in 1,000 km) that will be exceeded by 60% of the tires.

A. 44.66 B. 50 C. 55.81 D. 35.54 E. none of the above

33. [1] **Multiple Choice Question**

Suppose that the number of bad cheques received by a bank in one day is a Poisson random variable with mean $\lambda = 3$. Determine the probability that the bank will receive 4 bad cheques in 2 days.

A. 0.134 B. 0.058 C. 0.316 D. 0.205 E. none of the above

34. [1] **Multiple Choice Question**

Suppose the reaction time at a traffic light that a driver takes to brake is normally distributed with mean $\mu = 1.25$ seconds and standard deviation $\sigma = 0.2$ seconds. What is the probability that the time to react to a red traffic light will be larger than 1.75 seconds?

A. 0.1054 B. 0.2397 C. 0.3456 D. 0.0062 E. 0.0538

35. [1] **Multiple Choice Question**

The time (in days) that a surveillance camera will be used is a continuous random variable X with probability density function $f(x) = (10 - x)/50$, for $0 \leq x \leq 10$. What is $E[X]$?

A. 10/3 B. 5 C. 75/3 D. 10 E. none of the preceding

36. [1] **Multiple Choice Question**

Let X be a random variable with the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x \leq -1 \\ 0.75x - 0.25x^3 + 0.5 & -1 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$$

Compute the mean and the variance of X .

A. 0 and 3 B. 1 and 0.2 C. 1 and 3 D. 0 and 0.2 E. none of the preceding

37. [1] **Multiple Choice Question**

The cross section of a plastic tube for use in pulmonary resuscitators is normally distributed with a mean of 12.5mm^2 and a standard deviation of 0.2mm^2 . When the cross section is less than 12mm^2 or more than 13mm^2 , the tube will not be adjusted properly. What is the probability that a randomly selected tube will be adjusted properly?

A. 0.9538 B. 0.9998 C. 0.9890 D. 0.9745 E. 0.9876

38. [1] **Multiple Choice Question**

Consider the discrete random variables X and Y with the following joint probability mass function:

x	y	$f_{XY}(x, y)$
-1	0	1/8
0	-1	1/4
0	1	1/4
1	0	1/8
-1	1	1/8
1	-1	1/8

Compute the covariance $\text{Cov}(X, Y) = E[XY] - E[X]E[Y]$.

A. 7/8 B. 3/8 C. 1/4 D. -1/4 E. none of the preceding

39. [1] **Multiple Choice Question**

Consider the discrete random variables X and Y with the following joint probability mass function:

x	y	$f_{XY}(x, y)$
-1	0	1/8
0	-1	1/4
0	1	1/4
1	0	1/8
-1	1	1/8
1	-1	1/8

Given that X is not negative, what is the probability that Y is also not negative?

A. 0.5 B. 0.8 C. 0.4 D. 0.25 E. none of the preceding

40. [1] **Multiple Choice Question**

Consider the discrete random variables X and Y with the following joint probability mass function:

x	y	$f_{XY}(x, y)$
-1	0	1/8
0	-1	1/4
0	1	1/4
1	0	1/8
-1	1	1/8
1	-1	1/8

What are the mean and the variance of X ?

A. 0; 0.25 B. 0; 0.5 C. 0.1; 0.5 D. 0.5; 0.5 E. none of the preceding

41. [1] **Multiple Choice Question**

Consider the discrete random variables X and Y with the following joint probability mass function:

x	y	$f_{XY}(x, y)$
-1	0	1/8
0	-1	1/4
0	1	1/4
1	0	1/8
-1	1	1/8
1	-1	1/8

What is $P(X = 1|Y = 0)$? Are X and Y independent?

- A. 0; independent B. 1/2; independent C. 1/2; dependent D. 1/8; dependent
E. none of the preceding

42. [1] **Multiple Choice Question**

It is known that a particular company produces products of which 30% are defective. We select items at random and identify it as being defective or not. Calculate the probability that the sixth selected item will be the 3rd defective.

- A. 0.38282 B. 0.09261 C. 0.24518 D. 0.75494 E. none of the above

43. [1] **Multiple Choice Question**

Let X_1, \dots, X_7 be a random sample from a population with mean μ and variance σ^2 . Consider the following estimators for μ :

$$\hat{\Theta}_1 = \frac{X_1 + \dots + X_7}{7}, \quad \hat{\Theta}_2 = \frac{2X_1 - X_6 \dots + X_4}{2}.$$

Are these estimators unbiased (i.e. is their expectation equal to μ)?

- A. Both estimators are unbiased. B. Both estimators are biased.
C. Only the second is unbiased. D. Only the first is unbiased.
E. Insufficient information.

44. [1] **Multiple Choice Question**

Let X_1, \dots, X_7 be a random sample from a population with mean μ and variance σ^2 . Consider the following estimators for μ :

$$\hat{\Theta}_1 = \frac{X_1 + \dots + X_7}{7}, \quad \hat{\Theta}_2 = \frac{2X_1 - X_6 \dots + X_4}{2}.$$

Compute the variance of the estimators.

- A. $V[\hat{\Theta}_1] = \sigma^2/7$, $V[\hat{\Theta}_2] = 3\sigma^2/2$ B. $V[\hat{\Theta}_1] = 3\sigma^2/2$, $V[\hat{\Theta}_2] = 3\sigma^2/7$
C. $V[\hat{\Theta}_1] = \sigma^2/49$, $V[\hat{\Theta}_2] = 3\sigma^2/2$ D. $V[\hat{\Theta}_1] = \sigma^2/49$, $V[\hat{\Theta}_2] = 3\sigma^2/2$
E. none of the preceding

45. [1] **Multiple Choice Question**

Here is a random sample for the ignition time (in seconds) for 4 upholstery materials exposed to heat: 2.58, 2.52, 4.04, 2.20. Obtain a point estimate for the population standard deviation σ of the ignition time.

- A. 0.125 B. 2.835 C. 0.8205 D. 0.6732 E. none of the preceding

46. [1] **Multiple Choice Question**

Consider a random sample of size $n = 15$ from a population with mean $\mu = 20$ and standard deviation $\sigma = 6$. The sample mean and sample standard deviation are \bar{X} and S , respectively. Determine a such that

$$P\left(\frac{\bar{X} - 20}{S/\sqrt{15}} \leq a\right) = 0.05.$$

- A. -1.753 B. -1.761 C. 1.753 D. 1.761 E. -1.645

47. [1] **Multiple Choice Question**

We measured the compressive strength for $n = 16$ specimens of concrete. Using the mean and standard deviation and assuming a normal population, we computed the following confidence interval $[2271.7688, 2308.2312]$. This interval is a confidence interval for the mean compressive strength at a level of confidence of 90%. We are told the the sample mean is $\bar{x} = 2290$. What is the value of the sample standard deviation s ?

- A. 44.33 B. 41.6 C. 83.2 D. 54.38 E. none of the preceding

48. [1] **Multiple Choice Question**

Suppose that X_1, \dots, X_{81} are independent random variables with probability density function $f_X(x) = 0.5 \exp(-x/2)$, $x > 0$. Approximate $P(X_1 + \dots + X_{81} > 170)$.

- A. 0.67 B. 0.16 C. 0.33 D. 0.95 E. none of the preceding

49. [1] **Multiple Choice Question**

A random sample of size $n_1 = 16$ is selected from a normal population with mean 75 and variance 288. A second random sample of size $n_2 = 9$ is selected from a normal population with mean 80 and variance 162. Assume that the random samples are independent. Let \bar{X}_1 and \bar{X}_2 be the respective sample means. Find the probability that $\bar{X}_1 + \bar{X}_2$ is larger than 156.5.

- A. 0.5987 B. 0.4013 C. 0.6231 D. 0.4235 E. none of the preceding

50. [1] **Multiple Choice Question**

A company produces photocopiers. The mean lifetime of a photocopier is 6 years and the standard deviation of the lifetime is 2.5 years. Consider a random sample of 50 photocopiers. Approximate the probability that the mean lifetime of these 50 photocopiers will exceed 7 years.

- A. 0.0023 B. 0.9977 C. 0.6554 D. 0.3446 E. none of the preceding

51. [1] **Multiple Choice Question**

Consider two independent populations with the following means and variances: $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$. To estimate the difference $\theta = \mu_1 - \mu_2$, we use $\hat{\Theta} = \bar{X} - \bar{Y}$, where \bar{X} and \bar{Y} are the sample means from the respective populations, based on samples of sizes n_1, n_2 , respectively. Which of the following statements is true?

- A. $E[\hat{\Theta}] = \theta$ and $\text{Var}[\hat{\Theta}] = \sigma_1^2/n_1 + \sigma_2^2/n_2$
B. $E[\hat{\Theta}] \neq \theta$ and $\text{Var}[\hat{\Theta}] = \sigma_1^2/n_1 + \sigma_2^2/n_2$
C. $E[\hat{\Theta}] = \theta$ and $\text{Var}[\hat{\Theta}] = \sigma_1^2/n_1 - \sigma_2^2/n_2$
D. $E[\hat{\Theta}] \neq \theta$ and $\text{Var}[\hat{\Theta}] = \sigma_1^2/n_1 - \sigma_2^2/n_2$
E. none of the preceding

52. [1] **Multiple Choice Question**

The service time at a customer service counter has a mean of 176 seconds with a variance of 400. A random sample of 100 clients is selected. What is the approximate probability that the sample mean will fall between 175 and 177 seconds?

- A. 0.691 B. 0.383 C. 0.277 D. 0.288 E. none of the preceding

53. [1] **Multiple Choice Question**

Assume that the yearly rainfall in Winnipeg is normally distributed with mean 35.4 inches and standard deviation 4.2 inches. A university student is in Winnipeg for 4 years. Assume that yearly rainfall are independent. Let $D = X_1 - X_4$ be the difference of the rainfall in year 1 and in year 4. Compute the mean and the standard deviation of D , respectively.

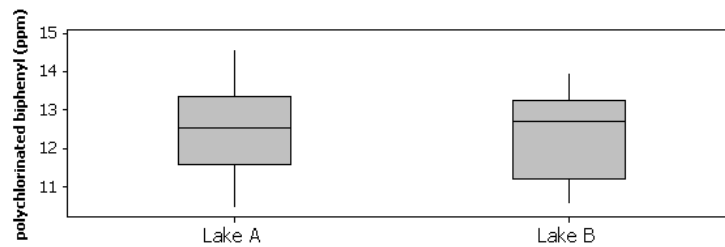
- A. 0; 8.4 B. 0; 5.9 C. 0; 0 D. -35.4; 5.9 E. -35.4; 8.4

54. [1] **Multiple Choice Question**

A sample of 20 fish are captured in lake *A* and their concentrations of polychlorinated biphenyl are measured using a certain technique. Furthermore, 15 fish are captured in lake *B* and their concentrations of polychlorinated biphenyl are measured. The data (in parts per million) are summarized as follows:

	Lake <i>A</i>	Lake <i>B</i>
mean	12.42	12.50
standard deviation	1.167	1.141
sample size	20	15

Boxplots are constructed for each lake:

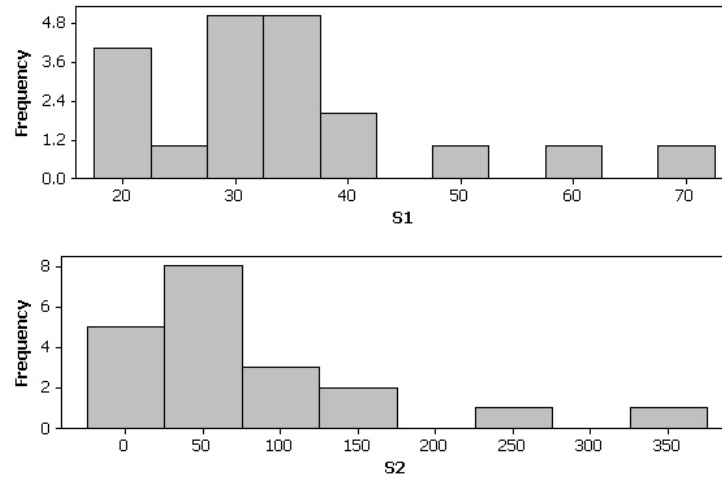


Is it reasonable to assume that

- A. the populations are normal with equal variances
- B. the populations are normal with unequal variances
- C. only one of the populations is normal
- D. the populations are not normal
- E. none of the preceding

55. [1] **Multiple Choice Question**

Suppose that we want to compare the waiting times between repairs of two systems (in days), S_1 and S_2 . Here are histograms of the waiting times:

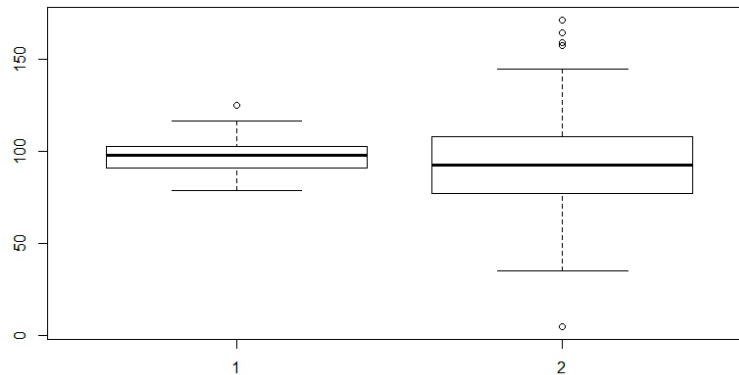


Is it reasonable to assume that

- A. the populations are normal with equal variances
- B. the populations are normal with unequal variances
- C. the populations are not normal
- D. only one of the populations is normal
- E. none of the preceding

56. [1] **Multiple Choice Question**

Consider the following boxplots:



It appears that the dispersion of

- A. the first sample is the largest
 - B. the second sample is the largest
 - C. the first and second sample is the same
 - D. both samples cannot be compared
 - E. none of the preceding
57. [1] **Multiple Choice Question**
 Consider the following random sample: 16, 13, 10, 17, 16, 14, 5, 12, 15, 8. Calculate the first quartile (choose the best answer).
- A. 9.5 B. 7.5 C. 10.5 D. 0.25 E. 11
58. [1] **Multiple Choice Question**
 Consider the following sample: 16, 13, 10, 17, 16, 14, 5, 12, 15, 8, 9, 10, 15, 22, 8. Calculate the mean and median of the sample, respectively.
- A. 11.67; 12.5 B. 190.0; 13 C. 12.667; 12.5 D. 12.667; 13
 - E. none of the preceding
59. [1] **Multiple Choice Question**
 Consider the following sample: 16, 13, 10, 17, 16, 14, 5, 12, 15, 8, 9, 10, 15, 22, 8. Calculate the variance and interquartile range of the sample, respectively.
- A. 19.38; 7 B. 15.7; 7 C. 4.40; 7 D. 19.38; 7.5 E. 19.38; 6.5
60. [1] **Multiple Choice Question**
 Consider the following random sample: 5, 34, 12, 10, 4. Calculate the mean and variance of the sample, respectively.
- A. 13; 149 B. 13; 119.2 C. 12; 119.2 D. 13; 12.2
 - E. none of the preceding

61. [1] **Multiple Choice Question**

Here are the waiting times (in minutes) for a bus for a particular person on 5 consecutive working days: 10, 1, 13, 9, 5 (presumably this was not happening in Ottawa...) Compute the mean and the variance of the sample, respectively.

- A. 7.6; 15.5 B. 7.6, 21.8 C. 6.9, 21.8 D. 6.9, 15.5
E. none of the preceding

62. [1] **Multiple Choice Question**

The city of Ottawa would like to know how many people are in favour of a property tax increase. Among 1000 residents, 300 replied "yes, I am in favour of a property tax increase." Determine a 95% confidence interval for the proportion of people that are in favour of a property tax increase.

- A. [0.202,0.254] B. [0.197,0.259] C. [0.194,0.262] D. [0.272,0.328]
E. none of the above

63. [1] **Multiple Choice Question**

A machine produces cylindrical metal pieces. A random sample of the pieces yields diameters

1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.04, 1.03, 1.01.

Determine a 99% confidence interval for the mean diameter. You may assume the diameters are normally distributed.

- A. [0.989, 1.022] B. [0.983, 1.035] C. [0.991, 1.034] D. [0.987, 0.024]
E. none of the above

64. [1] **Multiple Choice Question**

In a random sample of 100 houses in Halifax, 23 are heated by electricity. Construct a 90% confidence interval for the proportion of houses in Halifax that are heated by electricity.

- A. [0.1608, 0.2992] B. [0.1508, 0.2992] C. [0.0, 1.0] D. [0.1608, 0.3145]
E. none of the above

65. [1] **Multiple Choice Question**

An engineer measures the weight of steel pieces. The weights follow a normal distribution with known variance $\sigma^2 = 16$. The engineer wants to be 95% confident that the maximal error is at most $E = 0.2$ when estimating the mean. Determine the required sample size.

- A. 25 B. 1537 C. 423 D. 1083 E. none of the above

66. [1] **Multiple Choice Question**

A machine produces metal pieces which are cylindrical in shape. We select a sample of nine pieces and measure the diameters:

1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01, 1.03

The sample and sample standard deviation are $\bar{x} = 1.00556$ and $s = 0.02455$, respectively. Give a 95% confidence interval for the true mean diameter, assume that the population is normal.

- A. [0.989,1.022] B. [0.978,1.033] C. [0.991,1.034] D. [0.987,1.024]
E. none of the preceding

67. [1] **Multiple Choice Question**

Previous experience has shown that the fracture resistance of a wire used in the manufacture of drapery is normally distributed with $\sigma^2 = 2$. A random sample of 25 specimens have been examined and they yielded a mean resistance of $\bar{x} = 98$ psi. Give a 95% confidence interval for the mean resistance.

- A. [97.216,98.784] B. [97.216,98.554] C. [97.456,98.784] D. [97.446,98.554]
E. none of the preceding

68. [1] **Multiple Choice Question**

An expert wishes to determine the average time (in seconds) required to drill holes in some metal flange. Determine the required sample size to be 99% certain that the sample mean will be within 15 seconds of the true average, assuming that the standard deviation of the time is $\sigma = 30$ seconds.

- A. 26 B. 27 C. 28 D. 29 E. none of the preceding

69. [1] **Multiple Choice Question**

Let X equal the excess weight of a "1000 grams" bottle of soap. Assume that X follows a normal distribution with variance 169g^2 . What sample size is required to have a level of confidence of 95% that the maximum error of the estimate of the mean of the excess weight is less than 1.5g?

- A. 302 B. 287 C. 289 D. 301 E. 288

70. [1] **Multiple Choice Question**

Consider the observations of two different populations. The data collected have sample sizes $n_1 = 25$ and $n_2 = 29$. The mean of sample 1 is 18 and the mean of sample 2 is 16. Furthermore, the sample standard deviations are $s_1 = 5$ and $s_2 = 6$. For these data, the estimated standard error of the difference of the two means is:

- A. 11 B. 0.4 C. 1.5 D. 61 E. none of the preceding

71. [1] **Multiple Choice Question**

Previous experience has shown that the breaking strength of the fabric used in a certain brand of drapes is normally distributed with a standard deviation of 2 pounds per square inch. A random sample of 9 specimens is examined to reveal an average breaking strength of $\bar{x} = 98$ pounds per square inch. Determine the p -value required to test the hypothesis that the true mean exceeds 97.

A. 0.067 B. 0.012 C. 0.13 D. 0.006 E. none of the preceding

72. [1] **Multiple Choice Question**

Previous experience has shown that the breaking strength of the fabric used in a certain brand of drapes is normally distributed with a standard deviation of 2 pounds per square inch. A random sample of 9 specimens is examined to reveal an average breaking strength of $\bar{x} = 98$ pounds per square inch. Determine the p -value required to test the hypothesis that the true mean is not 97.

A. 0.067 B. 0.012 C. 0.13 D. 0.006 E. none of the preceding

73. [1] **Multiple Choice Question**

Previous experience has shown that the breaking strength of the fabric used in a certain brand of drapes is normally distributed with a standard deviation of 2 pounds per square inch. A random sample of 25 specimens is examined to reveal an average breaking strength of $\bar{x} = 98$ pounds per square inch. Determine the p -value required to test the hypothesis that the true mean is not 97.

A. 0.067 B. 0.012 C. 0.13 D. 0.006 E. none of the preceding

74. [1] **Multiple Choice Question**

Previous experience has shown that the breaking strength of the fabric used in a certain brand of drapes is normally distributed with with a mean of 97 pounds per square inch. A random sample of 9 specimens is examined to reveal an average breaking strength of $\bar{x} = 98$ pounds per square inch and a standard deviation of 2 pounds per square inch are observed. The p -value to test the hypothesis that the true mean is not 97 is between

A. (0.05,0.1) B. (0.1,0.2) C. (0.025,0.05) D. [0,0.025)
E. none of the preceding

75. [1] **Multiple Choice Question**

A new treatment has been developed for a certain type of cement, resulting in a mean pressure resistance of 5000kg/cm² and a standard deviation of 130kg/cm². To verify the null hypothesis of $\mu = 5000$ against the alternative of $\mu < 5000$, a random sample of 50 pieces of cement are examined. Determine the probability of committing a Type II error if $\mu = 4960$ and $\alpha = 0.05$.

A. 0.298 B. 0.954 C. 0.082 D. 0.105 E. none of the preceding

76. [1] **Multiple Choice Question**

An engineer measures the weights (in kilograms) of steel pieces. They would like to test $H_0 : \mu = 5$ against $H_1 : \mu > 5$. The weights follow a normal distribution with variance 16. Using a sample of size $n = 25$, the engineer decides to reject H_0 if $\bar{x} > 6$. Determine the probability of committing an error of Type I error.

A. 0.0500 B. 0.1057 C. 0.8943 D. 0.1000 E. none of the preceding

77. [1] **Multiple Choice Question**

An engineer measures the weights (in kilograms) of steel pieces. They would like to test $H_0 : \mu = 5$ against $H_1 : \mu > 5$. The weights follow a normal distribution with variance 16. Using a sample of size $n = 25$, the engineer decides to reject H_0 if $\bar{x} > 6$. Assuming that the true population mean is 5.2, determine the probability of committing an error of Type II error.

A. 0.8413 B. 0.05 C. 0.9332 D. 0.8943 E. none of the preceding

78. [1] **True or False Question**

A new treatment has been developed for a certain type of cement. To test the null hypothesis that $\mu = 5000$ against the alternative that $\mu \neq 5000$, where μ is the mean resistance in kg/cm², a sample of 50 samples of cement is examined and we observe $\bar{x} = 4775$ and $s = 130$. At $\alpha = 0.05$, the evidence against the null hypothesis in favour of the alternative hypothesis is significant.

True False

79. [1] **True or False Question**

An engineer measures the weights (in kilograms) of steel pieces. They would like to test $H_0 : \mu = 5$ against $H_1 : \mu > 5$. The weight of a steel piece is normally distributed. They select a random sample size of $n = 25$ steel pieces, and compute $\bar{x} = 6.7$ and $s = 2.37$. We cannot conclude that the mean weight is larger than 5 kg at a level of significance of 1%.

True False

80. [1] **True or False Question**

In order to establish a control chart for the mean of a process, 20 samples each of size 4 are collected. We note that $\sum_{i=1}^{20} \bar{x}_i = 4000$ and $\sum_{i=1}^{20} s_i = 500$. The value of the lower control limit of the chart for the mean is approximately equal to 195.5.

True False

81. [1] **True or False Question**

In order to establish a control chart for the mean of a process, 20 samples each of size 4 are collected. We note that $\sum_{i=1}^{20} \bar{x}_i = 4000$ and $\sum_{i=1}^{20} s_i = 500$. The value of the lower control limit of the chart for the mean is approximately equal to 183.

True False

82. [1] **True or False Question**

In order to establish a control chart for the mean of a process, 20 samples each of size 4 are collected. We note that $\sum_{i=1}^{20} \bar{x}_i = 4000$ and $\sum_{i=1}^{20} s_i = 500$. The value of the upper control limit of the chart for the mean is approximately equal to 217.

True False

83. [1] **True or False Question**

The length of time in minutes between consecutive calls to 911 in a small city has density

$$f(x) = \begin{cases} \frac{1}{20} e^{-x/20} & 0 < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

The probability that the time between consecutive calls is greater than 20 minutes is thus $1/e$.

True False

84. [1] **True or False Question**

A boiler has 4 relief valves which operate independently. The probability that each opens properly is 0.99. The probability that at least one opens properly is thus $1 - (0.99)^4$.

True False

85. [1] **True or False Question**

In order to establish a control chart for the mean of a process, 20 samples each of size 4 are collected. We note that $\sum_{i=1}^{20} \bar{x}_i = 4000$ and $\sum_{i=1}^{20} s_i = 500$. The value of the upper control limit of the chart for the mean is approximately equal to 204.5.

True False

86. [1] **True or False Question**

A random sample of 100 urban residents reveals that 50 believe in angels whereas in a random sample of 100 rural residents it is found that 65 believe in angels. We test the null hypothesis that the percentage of urban and rural residents who believe in angels is the same against the alternative that it is higher for rural residents. Based on this data, the corresponding test statistic is 2.165 and we reject the null hypothesis at a significance level $\alpha = 10\%$.

True False

87. [1] **True or False Question**

The percentage of males in 1986 who are 18-19 years old and married was 3.7%. To test whether or not this percentage had increased in 2015, a random sample of 300 males aged 18-19 was taken; 20 of which were married. Using a level of significance of $\alpha = 0.05$, we test the hypothesis H_0 that the proportion that is married is 3.7% against the alternative that it is larger. Based on the data, the observed value of the corresponding test statistic is 2.722 and we reject H_0 .

True False

88. [1] **True or False Question**

A postmix beverage machine is adjusted to release a certain amount of syrup into a chamber where it is mixed with carbonated water. A random sample of 25 beverages was found to have a mean syrup content of 1.05 fluid ounces and a standard deviation of 0.15 fluid ounces. Assume that the syrup content of a beverage is normally distributed. Then, to check the claim that the syrup content per beverage is about one ounce, we use the following test

$$H_0 : \mu = 1 \quad \text{against} \quad H_1 : \mu \neq 1.$$

Based on the samples, the p -value for the test is less than 0.05.

True False

89. [1] **True or False Question**

A candy maker produces mints whose weight follows a normal distribution with mean 21.37g and standard deviation 0.4g. Suppose 15 mints are selected at random. Let Y be the number of mints among them that weigh less than 20.857g. Then, $P(Y \leq 2) = 0.816$.

True False

90. [1] **True or False Question**

Students on a boat have 9 flags to arrange on a pole. There are 3 red, 4 yellow and 2 blue flags. Flags of the same colour are indistinguishable. A total of 1524 different signals can be sent by arranging all the 9 flags on the pole.

True False

91. [1] **True or False Question**

Let X, Y be independent random variables with $E[X] = E[Y] = 0$ and $\sigma_X = \sigma_Y = 5$. Then $\text{Var}\left(\frac{2X+3Y}{5}\right) = 1$.

True False

92. [1] **True or False Question**

Let X and Y be random variables with joint density function

$$f(x, y) = \begin{cases} 6y & 0 < y \leq x < 1 \\ 0 & \text{otherwise} \end{cases}$$

The marginal density of Y is $f_Y(y) = 3y(1 - y)$, for $0 < y < 1$.

True False

93. [1] **True or False Question**

The University of Ottawa is interested in offering its employees one of two employee benefit packages. A random sample of the university's employees is collected, and each person in the sample is asked to rate each of the two packages on an overall preference scale of 0 to 100. The results are

package A: 45, 67, 63, 50, 77, 60, 47, 39, 56, 68, 70

package B: 56, 79, 60, 45, 85, 39, 50, 41, 50, 69, 82

After analyzing the data, the University concludes that its employees prefer, on average, one package over the other, i.e., there is a significant difference between the two packages, at significance level $\alpha = 0.05$.

True False

94. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned} \sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731. \end{aligned}$$

What is an approximate 95% confidence interval for the slope of the line of best fit?

95. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned} \sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731. \end{aligned}$$

What is an approximate 95% confidence interval for the intercept of the line of best fit?

96. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned} \sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731. \end{aligned}$$

What is an approximate 99% confidence interval for the slope of the line of best fit?

97. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

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What is an approximate 99% confidence interval for the intercept of the line of best fit?

98. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned} \sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731. \end{aligned}$$

What is an approximate 95% confidence interval for the mean response at $x_0 = 60$?

99. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned} \sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731. \end{aligned}$$

What is an approximate 95% confidence interval for the mean response at $x_0 = 90$?

100. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned}\sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731.\end{aligned}$$

What is an approximate 99% confidence interval for the mean response at $x_0 = 90$?

101. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned}\sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731.\end{aligned}$$

What is an approximate 99% confidence interval for the mean response at $x_0 = 90$?

102. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned}\sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731.\end{aligned}$$

What is an approximate 95% prediction interval for the response y_0 at $x_0 = 60$?

103. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned}\sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731.\end{aligned}$$

What is an approximate 95% prediction interval for the response y_0 at $x_0 = 90$?

104. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned}\sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731.\end{aligned}$$

What is an approximate 99% prediction interval for the response y_0 at $x_0 = 60$?

105. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned}\sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731.\end{aligned}$$

What is an approximate 99% prediction interval for the response y_0 at $x_0 = 90$?

106. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned}\sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731.\end{aligned}$$

What is the mean squared error estimate for the variance of the residuals?

107. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\begin{aligned}\sum_{i=1}^n x_i &= 683, & \sum_{i=1}^n y_i &= 813, \\ \sum_{i=1}^n x_i^2 &= 47,405, & \sum_{i=1}^n x_i y_i &= 56,089, & \sum_{i=1}^n y_i^2 &= 66,731.\end{aligned}$$

What is the line of best fit for this data?

108. [2] **Short Answer Question**

We have a dataset with $n = 10$ pairs of observations (x_i, y_i) , and

$$\sum_{i=1}^n x_i = 683, \quad \sum_{i=1}^n y_i = 813,$$

$$\sum_{i=1}^n x_i^2 = 47,405, \quad \sum_{i=1}^n x_i y_i = 56,089, \quad \sum_{i=1}^n y_i^2 = 66,731.$$

What is the coefficient of correlation for this data?

109. [2] **Short Answer Question**

We have $m = 5$ preliminary samples of size $n = 3$ (some numbers have unfortunately been erased by accident by a clumsy co-op student):

i	$x_{i,1}$	$x_{i,2}$	$x_{i,3}$	\bar{x}_i	r_i	s_i
1	27.1	29.4		27.9		1.3
2	30.6	32.5	32.4	31.83	1.9	1.07
3	25.7	35.5	30	30.4		4.91
4	31.1	23.2	25	26.43	7.9	
5	24.1	34.2	27.4	28.57	10.1	5.15
total:				145.13	32	16.57

What is the control chart (give the interval) for \bar{X} from \bar{R} ?

110. [2] **Short Answer Question**

We have $m = 5$ preliminary samples of size $n = 3$ (some numbers have unfortunately been erased by accident by a clumsy co-op student):

i	$x_{i,1}$	$x_{i,2}$	$x_{i,3}$	\bar{x}_i	r_i	s_i
1	27.1	29.4		27.9		1.3
2	30.6	32.5	32.4	31.83	1.9	1.07
3	25.7	35.5	30	30.4		4.91
4	31.1	23.2	25	26.43	7.9	
5	24.1	34.2	27.4	28.57	10.1	5.15
total:				145.13	32	16.57

What is the control chart (give the interval) for \bar{X} from \bar{S} ?

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>									
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90
8	0	0.4305	0.1678	0.1001	0.0576	0.0168	0.0039	0.0007	0.0001	0.0000	
	1	0.8131	0.5033	0.3671	0.2553	0.1064	0.0352	0.0085	0.0013	0.0001	
	2	0.9619	0.7969	0.6785	0.5518	0.3154	0.1445	0.0498	0.0113	0.0012	0.0000
	3	0.9950	0.9437	0.8862	0.8059	0.5941	0.3633	0.1737	0.0580	0.0104	0.0004
	4	0.9996	0.9896	0.9727	0.9420	0.8263	0.6367	0.4059	0.1941	0.0563	0.0050
	5	1.0000	0.9988	0.9958	0.9887	0.9502	0.8555	0.6846	0.4482	0.2031	0.0381
	6		0.9999	0.9996	0.9987	0.9915	0.9648	0.8936	0.7447	0.4967	0.1869
	7		1.0000	1.0000	0.9999	0.9993	0.9961	0.9832	0.9424	0.8322	0.5695
8					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
9	0	0.3874	0.1342	0.0751	0.0404	0.0101	0.0020	0.0003	0.0000		
	1	0.7748	0.4362	0.3003	0.1960	0.0705	0.0195	0.0038	0.0004	0.0000	
	2	0.9470	0.7382	0.6007	0.4628	0.2318	0.0898	0.0250	0.0043	0.0003	0.0000
	3	0.9917	0.9144	0.8343	0.7297	0.4826	0.2539	0.0994	0.0253	0.0031	0.0001
	4	0.9991	0.9804	0.9511	0.9012	0.7334	0.5000	0.2666	0.0988	0.0196	0.0009
	5	0.9999	0.9969	0.9900	0.9747	0.9006	0.7461	0.5174	0.2703	0.0856	0.0083
	6	1.0000	0.9997	0.9987	0.9957	0.9750	0.9102	0.7682	0.5372	0.2618	0.0530
	7		1.0000	0.9999	0.9996	0.9962	0.9805	0.9295	0.8040	0.5638	0.2252
	8			1.0000	1.0000	0.9997	0.9980	0.9899	0.9596	0.8658	0.6126
9					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
10	0	0.3487	0.1074	0.0563	0.0282	0.0060	0.0010	0.0001	0.0000		
	1	0.7361	0.3758	0.2440	0.1493	0.0464	0.0107	0.0017	0.0001	0.0000	
	2	0.9298	0.6778	0.5256	0.3828	0.1673	0.0547	0.0123	0.0016	0.0001	
	3	0.9872	0.8791	0.7759	0.6496	0.3823	0.1719	0.0548	0.0106	0.0009	0.0000
	4	0.9984	0.9672	0.9219	0.8497	0.6331	0.3770	0.1662	0.0473	0.0064	0.0001
	5	0.9999	0.9936	0.9803	0.9527	0.8338	0.6230	0.3669	0.1503	0.0328	0.0016
	6	1.0000	0.9991	0.9965	0.9894	0.9452	0.8281	0.6177	0.3504	0.1209	0.0128
	7		0.9999	0.9996	0.9984	0.9877	0.9453	0.8327	0.6172	0.3222	0.0702
	8		1.0000	1.0000	0.9999	0.9983	0.9893	0.9536	0.8507	0.6242	0.2639
	9				1.0000	0.9999	0.9990	0.9940	0.9718	0.8926	0.6513
10					1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	
11	0	0.3138	0.0859	0.0422	0.0198	0.0036	0.0005	0.0000			
	1	0.6974	0.3221	0.1971	0.1130	0.0302	0.0059	0.0007	0.0000		
	2	0.9104	0.6174	0.4552	0.3127	0.1189	0.0327	0.0059	0.0006	0.0000	
	3	0.9815	0.8389	0.7133	0.5696	0.2963	0.1133	0.0293	0.0043	0.0002	
	4	0.9972	0.9496	0.8854	0.7897	0.5328	0.2744	0.0994	0.0216	0.0020	0.0000
	5	0.9997	0.9883	0.9657	0.9218	0.7535	0.5000	0.2465	0.0782	0.0117	0.0003
	6	1.0000	0.9980	0.9924	0.9784	0.9006	0.7256	0.4672	0.2103	0.0504	0.0028
	7		0.9998	0.9988	0.9957	0.9707	0.8867	0.7037	0.4304	0.1611	0.0185
	8		1.0000	0.9999	0.9994	0.9941	0.9673	0.8811	0.6873	0.3826	0.0896
	9			1.0000	1.0000	0.9993	0.9941	0.9698	0.8870	0.6779	0.3026
	10					1.0000	0.9995	0.9964	0.9802	0.9141	0.6862
11						1.0000	1.0000	1.0000	1.0000	1.0000	

Table A.1 (continued) Binomial Probability Sums $\sum_{x=0}^r b(x; n, p)$

<i>n</i>	<i>r</i>	<i>p</i>										
		0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	
12	0	0.2824	0.0687	0.0317	0.0138	0.0022	0.0002	0.0000				
	1	0.6590	0.2749	0.1584	0.0850	0.0196	0.0032	0.0003	0.0000			
	2	0.8891	0.5583	0.3907	0.2528	0.0834	0.0193	0.0028	0.0002	0.0000		
	3	0.9744	0.7946	0.6488	0.4925	0.2253	0.0730	0.0153	0.0017	0.0001		
	4	0.9957	0.9274	0.8424	0.7237	0.4382	0.1938	0.0573	0.0095	0.0006	0.0000	
	5	0.9995	0.9806	0.9456	0.8822	0.6652	0.3872	0.1582	0.0386	0.0039	0.0001	
	6	0.9999	0.9961	0.9857	0.9614	0.8418	0.6128	0.3348	0.1178	0.0194	0.0005	
	7	1.0000	0.9994	0.9972	0.9905	0.9427	0.8062	0.5618	0.2763	0.0726	0.0043	
	8		0.9999	0.9996	0.9983	0.9847	0.9270	0.7747	0.5075	0.2054	0.0256	
	9		1.0000	1.0000	0.9998	0.9972	0.9807	0.9166	0.7472	0.4417	0.1109	
	10				1.0000	0.9997	0.9968	0.9804	0.9150	0.7251	0.3410	
	11					1.0000	0.9998	0.9978	0.9862	0.9313	0.7176	
12						1.0000	1.0000	1.0000	1.0000	1.0000		
13	0	0.2542	0.0550	0.0238	0.0097	0.0013	0.0001	0.0000				
	1	0.6213	0.2336	0.1267	0.0637	0.0126	0.0017	0.0001	0.0000			
	2	0.8661	0.5017	0.3326	0.2025	0.0579	0.0112	0.0013	0.0001			
	3	0.9658	0.7473	0.5843	0.4206	0.1686	0.0461	0.0078	0.0007	0.0000		
	4	0.9935	0.9009	0.7940	0.6543	0.3530	0.1334	0.0321	0.0040	0.0002		
	5	0.9991	0.9700	0.9198	0.8346	0.5744	0.2905	0.0977	0.0182	0.0012	0.0000	
	6	0.9999	0.9930	0.9757	0.9376	0.7712	0.5000	0.2288	0.0624	0.0070	0.0001	
	7	1.0000	0.9988	0.9944	0.9818	0.9023	0.7095	0.4256	0.1654	0.0300	0.0009	
	8		0.9998	0.9990	0.9960	0.9679	0.8666	0.6470	0.3457	0.0991	0.0065	
	9		1.0000	0.9999	0.9993	0.9922	0.9539	0.8314	0.5794	0.2527	0.0342	
	10			1.0000	0.9999	0.9987	0.9888	0.9421	0.7975	0.4983	0.1339	
	11				1.0000	0.9999	0.9983	0.9874	0.9363	0.7664	0.3787	
	12					1.0000	0.9999	0.9987	0.9903	0.9450	0.7458	
13						1.0000	1.0000	1.0000	1.0000	1.0000		
14	0	0.2288	0.0440	0.0178	0.0068	0.0008	0.0001	0.0000				
	1	0.5846	0.1979	0.1010	0.0475	0.0081	0.0009	0.0001				
	2	0.8416	0.4481	0.2811	0.1608	0.0398	0.0065	0.0006	0.0000			
	3	0.9559	0.6982	0.5213	0.3552	0.1243	0.0287	0.0039	0.0002			
	4	0.9908	0.8702	0.7415	0.5842	0.2793	0.0898	0.0175	0.0017	0.0000		
	5	0.9985	0.9561	0.8883	0.7805	0.4859	0.2120	0.0583	0.0083	0.0004		
	6	0.9998	0.9884	0.9617	0.9067	0.6925	0.3953	0.1501	0.0315	0.0024	0.0000	
	7	1.0000	0.9976	0.9897	0.9685	0.8499	0.6047	0.3075	0.0933	0.0116	0.0002	
	8		0.9996	0.9978	0.9917	0.9417	0.7880	0.5141	0.2195	0.0439	0.0015	
	9		1.0000	0.9997	0.9983	0.9825	0.9102	0.7207	0.4158	0.1298	0.0092	
	10			1.0000	0.9998	0.9961	0.9713	0.8757	0.6448	0.3018	0.0441	
	11				1.0000	0.9994	0.9935	0.9602	0.8392	0.5519	0.1584	
	12					0.9999	0.9991	0.9919	0.9525	0.8021	0.4154	
	13					1.0000	0.9999	0.9992	0.9932	0.9560	0.7712	
14						1.0000	1.0000	1.0000	1.0000	1.0000		

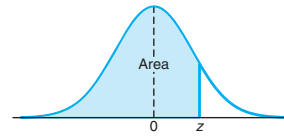


Table A.3 Areas under the Normal Curve

<i>z</i>	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

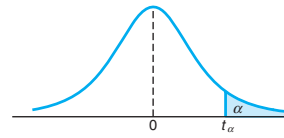


Table A.4 Critical Values of the *t*-Distribution

<i>v</i>	α						
	0.40	0.30	0.20	0.15	0.10	0.05	0.025
1	0.325	0.727	1.376	1.963	3.078	6.314	12.706
2	0.289	0.617	1.061	1.386	1.886	2.920	4.303
3	0.277	0.584	0.978	1.250	1.638	2.353	3.182
4	0.271	0.569	0.941	1.190	1.533	2.132	2.776
5	0.267	0.559	0.920	1.156	1.476	2.015	2.571
6	0.265	0.553	0.906	1.134	1.440	1.943	2.447
7	0.263	0.549	0.896	1.119	1.415	1.895	2.365
8	0.262	0.546	0.889	1.108	1.397	1.860	2.306
9	0.261	0.543	0.883	1.100	1.383	1.833	2.262
10	0.260	0.542	0.879	1.093	1.372	1.812	2.228
11	0.260	0.540	0.876	1.088	1.363	1.796	2.201
12	0.259	0.539	0.873	1.083	1.356	1.782	2.179
13	0.259	0.538	0.870	1.079	1.350	1.771	2.160
14	0.258	0.537	0.868	1.076	1.345	1.761	2.145
15	0.258	0.536	0.866	1.074	1.341	1.753	2.131
16	0.258	0.535	0.865	1.071	1.337	1.746	2.120
17	0.257	0.534	0.863	1.069	1.333	1.740	2.110
18	0.257	0.534	0.862	1.067	1.330	1.734	2.101
19	0.257	0.533	0.861	1.066	1.328	1.729	2.093
20	0.257	0.533	0.860	1.064	1.325	1.725	2.086
21	0.257	0.532	0.859	1.063	1.323	1.721	2.080
22	0.256	0.532	0.858	1.061	1.321	1.717	2.074
23	0.256	0.532	0.858	1.060	1.319	1.714	2.069
24	0.256	0.531	0.857	1.059	1.318	1.711	2.064
25	0.256	0.531	0.856	1.058	1.316	1.708	2.060
26	0.256	0.531	0.856	1.058	1.315	1.706	2.056
27	0.256	0.531	0.855	1.057	1.314	1.703	2.052
28	0.256	0.530	0.855	1.056	1.313	1.701	2.048
29	0.256	0.530	0.854	1.055	1.311	1.699	2.045
30	0.256	0.530	0.854	1.055	1.310	1.697	2.042
40	0.255	0.529	0.851	1.050	1.303	1.684	2.021
60	0.254	0.527	0.848	1.045	1.296	1.671	2.000
120	0.254	0.526	0.845	1.041	1.289	1.658	1.980
∞	0.253	0.524	0.842	1.036	1.282	1.645	1.960

Control Chart Constants

n	A_3	B_3	B_4	A_2	D_3	D_4
4	1.63	0	2.27	0.73	0	2.28
5	1.43	0	2.09	0.58	0	2.11
6	1.29	0.03	1.97	0.48	0	2.00
8	1.10	0.185	1.815	0.37	0.14	1.86
10	0.98	0.28	1.72	0.31	0.22	1.78
20	0.68	0.51	1.49	0.18	0.41	1.59

Sample Size = n	A_2	A_3
2	1.880	2.659
3	1.023	1.954
4	0.729	1.628
5	0.577	1.427
6	0.483	1.287
7	0.419	1.182
8	0.373	1.099
9	0.337	1.032
10	0.308	0.975
11	0.285	0.927
12	0.266	0.886
13	0.249	0.850
14	0.235	0.817
15	0.223	0.789
16	0.212	0.763
17	0.203	0.739
18	0.194	0.718
19	0.187	0.698
20	0.180	0.680
21	0.173	0.663
22	0.167	0.647
23	0.162	0.633
24	0.157	0.619
25	0.153	0.606