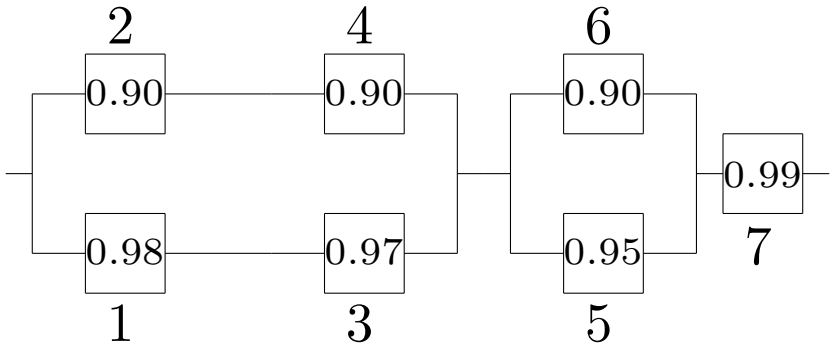
Q17. The following system operates only if there is a path of functional device from left to the right. The probability that each device functions is as shown. What is the probability that the circuit operates? Assume independence.



Solution: let Box A consist of components $1, 2, 3, 4$; Box B of components $5, 6$; Box C of component 7. Since all components are independent,

$$P(\text{system works}) = P(A \text{ works})P(B \text{ works})P(7 \text{ works}).$$

Now, B is just a parallel system, so that it works if any of its two components work:

$$P(B \text{ works}) = 0.9 + 0.95 - 0.9 \cdot 0.95 = 1.85 - 0.855 = 0.995.$$

Furthermore, since all the components are independent of one another,

$$P(2 \text{ and } 4 \text{ work}) = P(2 \text{ works})P(4 \text{ works}) = 0.9 \cdot 0.9 = 0.81$$
$$P(1 \text{ and } 3 \text{ work}) = P(1 \text{ works})P(3 \text{ works}) = 0.98 \cdot 0.97 = 0.9506.$$

Now, A is a parallel system consisting of two independent sub-systems: $2, 4$ and $1, 3$, so that

$$\begin{aligned} P(A \text{ works}) &= P(2 \text{ and } 4 \text{ work}) + P(1 \text{ and } 3 \text{ work}) \\ &\quad - P(2 \text{ and } 4 \text{ work})P(1 \text{ and } 3 \text{ work}) \\ &= 0.81 + 0.9506 - 0.81 \cdot 0.9506 = 0.9906. \end{aligned}$$

Thus

$$P(\text{system works}) = 0.9906 \cdot 0.995 \cdot 0.99 = 0.9758.$$