Q23. A student has 5 blue marbles and 4 white marbles in his left pocket, and 4 blue marbles and 5 white marbles in his right pocket. If they transfer one marble at random from their left pocket to his right pocket, what is the probability of them then drawing a blue marble from their right pocket?

Solution: for notation, let BL, BR, and WL denote drawing a blue marble from the left pocket, a blue marble from the right pocket, and a white marble from the left pocket, respectively. By the Law of Total Probability,

$$\begin{aligned} P(BR) &= P(BL \cap BR) + P(WL \cap BR) \\ &= P(BL)P(BR|BL) + P(WL)P(BR|WL) \\ &= \frac{5}{9} \cdot \frac{5}{10} + \frac{4}{9} \cdot \frac{4}{10} = \frac{41}{90} \approx 0.456. \end{aligned}$$

Q25. An urn contains four balls numbered 1 through 4. The balls are selected one at a time, without replacement. A match occurs if ball m is the mth ball selected. Let the event A_i denote a match on the ith draw, $i = 1, 2, 3, 4$.

a) Compute $P(A_i)$, $i = 1, 2, 3, 4$.

b) Compute $P(A_i \cap A_j)$, $i, j = 1, 2, 3, 4$, $i \neq j$.

c) Compute $P(A_i \cap A_j \cap A_k)$, $i, j, k = 1, 2, 3, 4$, $i \neq j, i \neq k, j \neq k$.

d) What is the probability of at least 1 match?

Solution: (difficult!)

a) $P(A_i) = \frac{3!}{4!}$.

b) $P(A_i \cap A_j) = \frac{2!}{4!}$.

c) $P(A_i \cap A_j \cap A_k) = \frac{1!}{4!}$.

d) $P(A_1 \cup A_2 \cup A_3 \cup A_4) = \frac{1}{1!} - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!}$.

Q26. The probability that a company's workforce has at least one accident in a given month is $(0.01)k$, where k is the number of days in the month. Assume that the number of accidents is independent from month to month. If the company's year starts on January 1, what is the probability that the first accident occurs in April?

Solution: for any month X, let X represent the event that an accident takes place during month X. If the monthly probabilities are independent of one another, then

$$\begin{aligned} P(J^c \cap F^c \cap M^c \cap A) &= P(J^c)P(F^c)P(M^c)P(A) \\ &= (1 - 31(0.01)) \cdot (1 - 28(0.01)) \cdot (1 - 31(0.01)) \cdot 30(0.01) \\ &= 0.69 \cdot 0.72 \cdot 0.69 \cdot 0.30 \approx 0.103. \end{aligned}$$