# MAT 2377
# Probability and Statistics for Engineers

## Chapter 5
## Point and Interval Estimation

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# Contents

# 5.1 – Statistical Inference

One of the goals of **statistical inference** is to draw conclusions about a **population** based on a random sample from the population.

**Examples:**

- Can we assess the reliability of a product's manufacturing process by randomly selecting a sample of the final product and determining how many of them are compliant according to some quality assessment scheme?

- Can we determine who will win an election by polling a small sample of respondents?

Specifically, we seek to estimate an unknown **parameter** $\theta$, say, using a single quantity called the **point estimate** $\hat{\theta}$.

This point estimate is obtained using a **statistic**, which is simply a function of a random sample. The probability distribution of the statistic is its **sampling distribution**. Describing these is a main research avenue.

**Example:** consider a process that manufactures gear wheels (in some standard gauge). Let $X$ be the random variable that records the weight of a randomly selected gear wheel. What is the population mean $\mu_X = \mathrm{E}[X]$?.

**Solution:** in the absence of $f(x)$, we can estimate $\mu = X$ with the help of a random sample $X_1, \ldots, X_n$ of gear wheel weight measurements, $via$ the sample mean statistic:

$$\overline{X} = \frac{X_1 + \cdots + X_n}{n}, \quad \text{which is } \approx \mathcal{N}\left(\mu, \sigma^2/n\right) \text{ according to C.L.T.}$$

# Statistics

Examples of statistics include:

- sample mean and sample median
- sample variance and sample standard deviation
- sample quantiles (median, quartiles, percentiles)
- test statistics ($t-$statistics, $\chi^2-$statistics, $f-$statistics, etc.)
- order statistics (sample maximum and minimum, sample range, etc.)
- sample moments and functions thereof (skewness, kurtosis, etc.)

# Estimator Variance and Standard Error

The **standard error** of a statistic is the **standard deviation of its sampling distribution**.

For instance, if observations $X_1, \ldots, X_n$ come from a a population with **unknown mean** $\mu$ and **known variance** $\sigma^2$, then $\mathrm{Var}(\overline{X}) = \sigma^2/n$ and the **standard error of** $\overline{X}$ is

$$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}.$$

If the variance of the original population is **unknown**, then it is estimated by the sample variance $S^2$ and the **estimated standard error of** $\overline{X}$ is

$$\hat{\sigma}_{\overline{X}} = \frac{S}{\sqrt{n}}, \quad \text{where} \quad S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

## Examples:

1. A sample of 20 baseball player heights (in inches) is shown below.

   `74,74,72,72,73,69,69,71,76,71,73,73,74,74,69,70,72,73,75,78.`

   Let $\overline{X}$ be the sampling mean of the heights. Then,

   $$\overline{X} = \frac{X_1 + \cdots + X_{20}}{20} = 72.6$$

   and the sample variance $S^2$ is

   $$S^2 = \frac{1}{20-1}\sum_{i=1}^{20}(X_i - 72.6)^2 \approx 5.6211.$$

The standard error of $\overline{X}$ is thus

$$\hat{\sigma}_{\overline{X}} = \frac{S}{\sqrt{20}} \approx \sqrt{\frac{5.6211}{20}} \approx 0.5301.$$

2. Consider a sample $\{X_1, \ldots, X_{100}\}$ of independent observations selected from a normal population $\mathcal{N}(\mu, \sigma^2)$ where $\sigma = 50$ is known, but $\mu$ is not. What is the best estimate of $\mu$? What is the sampling distribution of that estimate?

   **Solution:** the sample mean $\overline{X} = \frac{X_1 + \cdots + X_{100}}{100}$ provides the best estimate of $\mu_X = \mu_{\overline{X}}$. The standard error of $\overline{X}$ is $\sigma_{\overline{X}} = \frac{50}{\sqrt{100}} = 5$. Since the observations are sampled independently from a normal population with mean $\mu$ and standard deviation $50$, $\overline{X} \sim \mathcal{N}(\mu, 5^2) = \mathcal{N}(\mu, 25)$, according to the CLT.

# 5.2 – C.I. for $\mu$ when $\sigma$ is Known

Consider a sample $\{x_1, \ldots, x_n\}$ from a normal population with **known** variance $\sigma^2$ and **unknown** mean $\mu$. The sample mean

$$\overline{x} = \frac{x_1 + \cdots + x_n}{n}$$

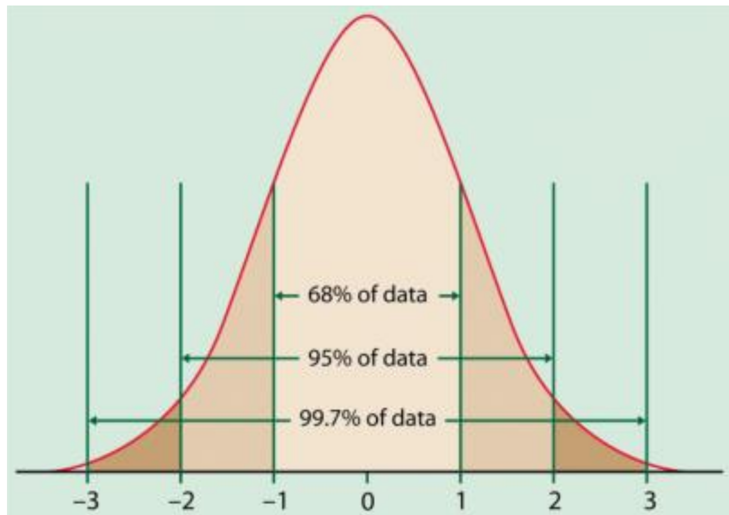
is a **point estimate** of $\mu$.

Of course, this estimate is not exact, because $\overline{x}$ is an observed value of $\overline{X}$; it is unlikely that the observed value $\overline{x}$ should coincide with $\mu$.

We know that $\overline{X} \sim \mathcal{N}(\mu, \sigma^2/n)$, so that

$$\frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathcal{N}(0, 1).$$

# The $68 - 96 - 99.7$ Rule and Confidence Intervals




$$P(-1 < Z < 1) \approx 0.683$$
$$P(-2 < Z < 2) \approx 0.955$$
$$P(-3 < Z < 3) \approx 0.997.$$

Whenever we observe a sample mean $\overline{X}$ from a normal population with mean $\mu$, we would expect the inequality

$$-k < Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} < k$$

to hold approximately

$$g(k) = \begin{cases} 68.3\% \text{ of the time} & \text{if } k = 1 \\ 95.5\% \text{ of the time} & \text{if } k = 2 \\ 99.7\% \text{ of the time} & \text{if } k = 3 \end{cases}$$

Equivalently, the **symmetric** $g(k)$ **confidence interval for** $\mu$ is

$$\overline{X} - k\frac{\sigma}{\sqrt{n}} < \mu < \overline{X} + k\frac{\sigma}{\sqrt{n}} \quad \Longrightarrow \quad \overline{X} \pm k\frac{\sigma}{\sqrt{n}}.$$

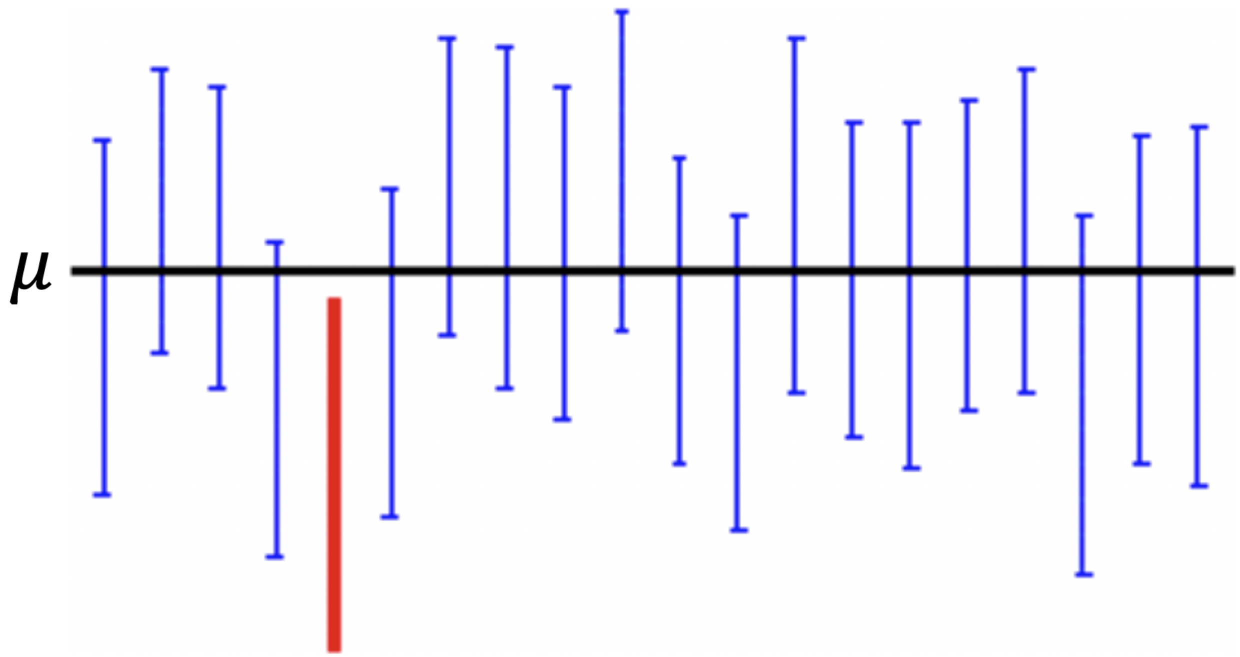
# Examples:

1. Consider a sample $\{X_1, \ldots, X_{64}\}$ from a normal population with standard deviation $\sigma = 72$ and unknown mean $\mu$. The sample mean is $\overline{X} = 375.2$. Build a symmetric $68.3\%$ confidence interval for $\mu$.

   **Solution:** according to the formula, the symmetric $68.3\%$ confidence interval $(k = 1)$ for $\mu$ in this situation is

$$375.2 \pm 1 \cdot \frac{72}{\sqrt{64}} \implies (375.2 - 9, 375.2 + 9) = (366.2, 384.2).$$

   **IMPORTANT:** this does not say that we're $68.3\%$ sure that the true $\mu$ is between $366.2$ and $384.2$. What it says is that when a sample of size $64$ is taken from a normal population $\mathcal{N}(\mu, 72^2)$ and a symmetric $68.3\%$ confidence interval for $\mu$ is built, $\mu$ will fall between the endpoints of the interval about $68.3\%$ of the time.




A $95\%$ C.I. indicates that we would expect $19$ out of $20$ samples from the same population to produce confidence intervals that contain the population parameter of interest, on average.

2. Build a symmetric $95.5\%$ confidence interval for $\mu$.

   **Solution:** the same formula applies, with $k = 2$.

$$375.2 \pm 2 \cdot \frac{72}{\sqrt{64}} \implies (375.2 - 18, 375.2 + 18) = (357.2, 393.2).$$

3. Build a symmetric $99.7\%$ confidence interval for $\mu$.

   **Solution:** the same formula applies, with $k = 3$.

$$375.2 \pm 3 \cdot \frac{72}{\sqrt{64}} \implies (375.2 - 27, 375.2 + 27) = (348.2, 402.2).$$

# C.I. for $\mu$ when $\sigma$ is Known (reprise)

Another approach to C.I. building is to specify the proportion of the area under $\phi(z)$ of interest, and then to determine the critical values (the endpoints) of the interval.

Let $\{X_1, \ldots, X_n\}$ be drawn from $N(\mu, \sigma^2)$. Recall that $\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \sim \mathcal{N}(0,1)$. For a **symmetric** $95\%$ **confidence interval**, we need to find $z^* > 0$ such that $P(-z^* < Z < z^*) \approx 0.95$.

But the LHS can be re-written as

$$\begin{aligned} P(-z^* < Z < z^*) &= \Phi(z^*) - \Phi(-z^*) \\ &= \Phi(z^*) - (1 - \Phi(z^*)) = 2\Phi(z^*) - 1 \end{aligned}$$

So we are looking for $z^*$ such that

$$0.95 = 2\Phi(z^*) - 1 \Longrightarrow \Phi(z^*) = \frac{0.95 + 1}{2} = 0.975.$$

From the normal table, we see that $\Phi(1.96) \approx 0.9750$, so that

$$P(-1.96 < Z < 1.96) = P\left(-1.96 < \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} < 1.96\right) \approx 0.95.$$

In other words, the inequality

$$-1.96 < \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} < 1.96$$

holds with probability $0.95$ (with the interpretation provided in Example 1).

Equivalently,

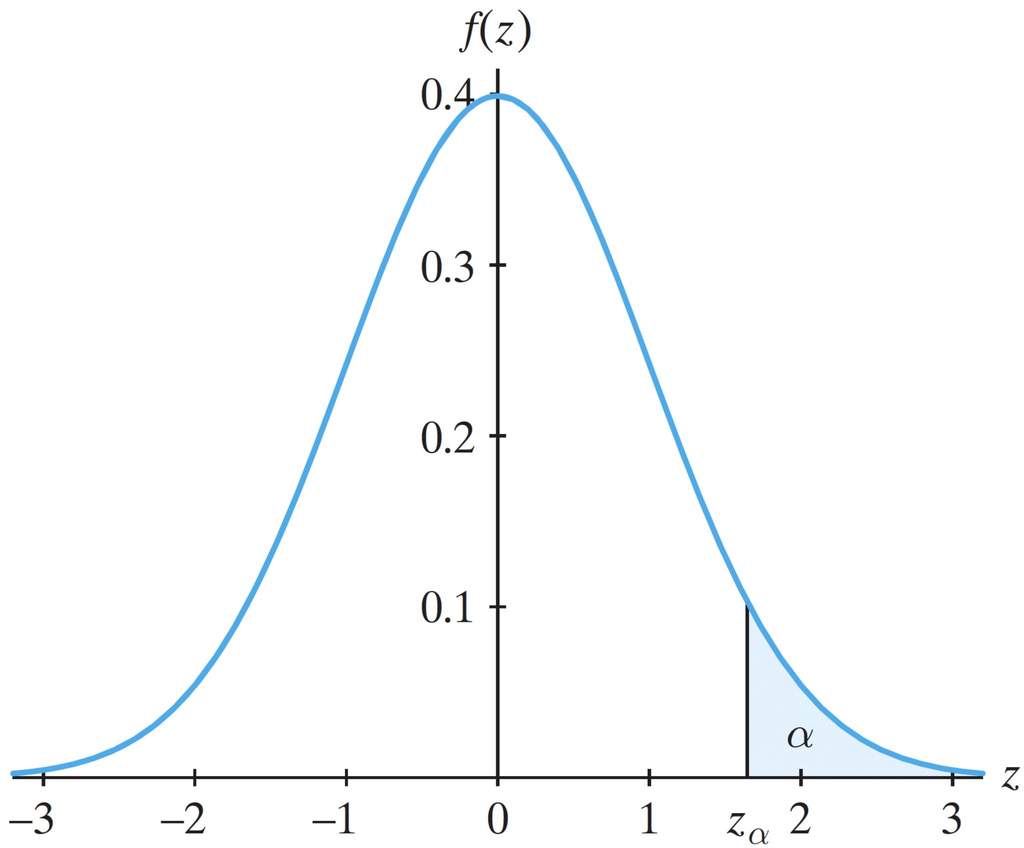
$$\overline{X} - 1.96\frac{\sigma}{\sqrt{n}} < \mu < \overline{X} + 1.96\frac{\sigma}{\sqrt{n}} \implies \overline{X} \pm 1.96\frac{\sigma}{\sqrt{n}}$$

is the **symmetric** $95\%$ **confidence interval for** $\mu$ **when** $\sigma$ **is known**.

A similar argument shows that

$$\overline{X} - 2.575\frac{\sigma}{\sqrt{n}} < \mu < \overline{X} + 2.575\frac{\sigma}{\sqrt{n}} \implies \overline{X} \pm 2.575\frac{\sigma}{\sqrt{n}}$$

is the **symmetric** $99\%$ **confidence interval for** $\mu$ **when** $\sigma$ **is known**.




$$P(Z > z_\alpha) = \alpha$$

$$P(Z > z) = 1 - \Phi(z) = \Phi(-z)$$

The **confidence level** $1-\alpha$ is usually expressed in terms of a **small** $\alpha$, e.g. $\alpha = 0.05 \Rightarrow 1-\alpha = 0.95$ confidence level.

For $\alpha = 0.01, 0.02, \ldots, 0.98, 0.99$, the corresponding $z_\alpha$ are called the **percentiles** of the standard normal distribution. In general,

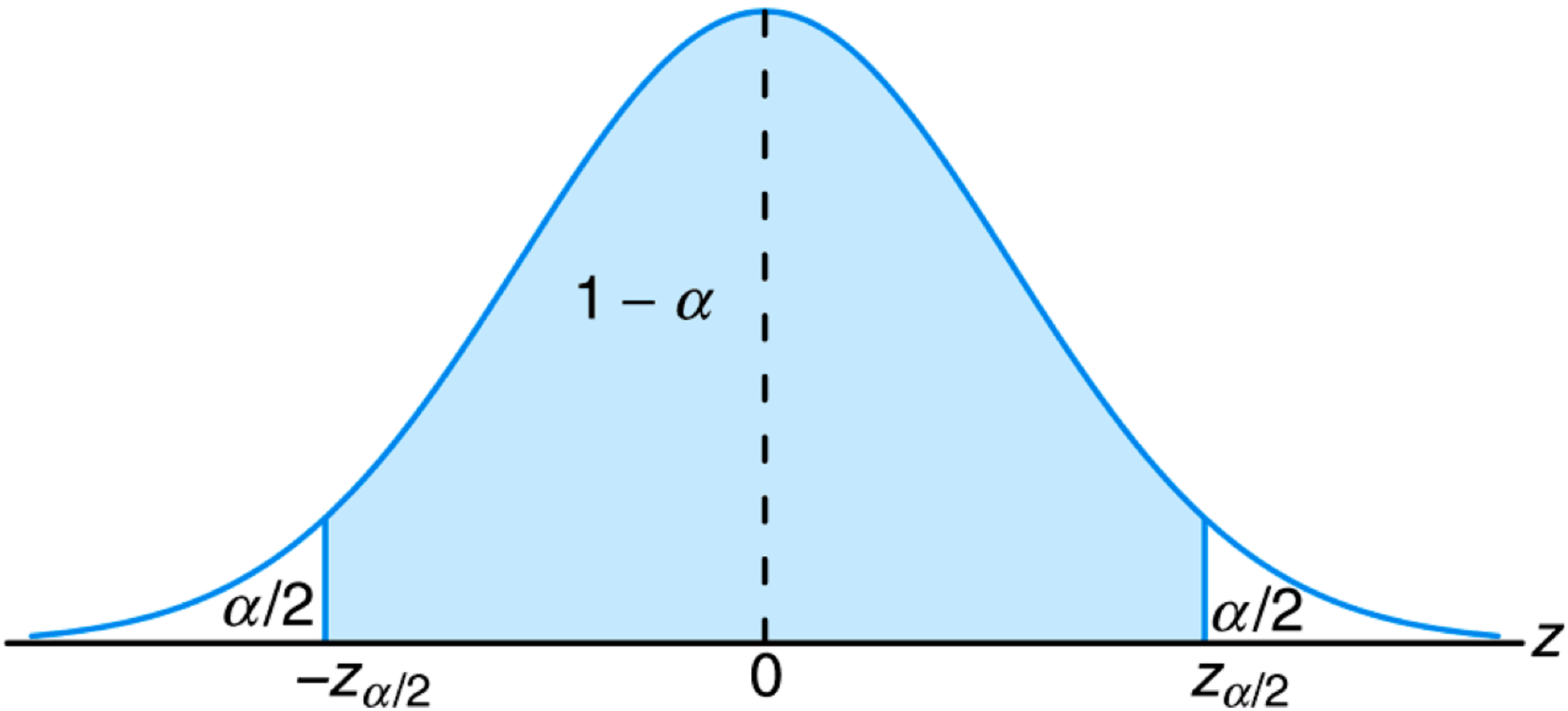
$$P(Z > z_\alpha) = \alpha \implies z_\alpha \text{ is the } 100(1-\alpha) \text{ percentile.}$$

For $2-$**sided confidence intervals**, the appropriate numbers are found by solving $P(|Z| > z^*) = \alpha$ for $z^*$. By the properties of $\mathcal{N}(0,1)$,

$$\alpha = P(|Z| > z^*) = 1 - P(-z^* < Z < z^*) = 1 - (2\Phi(z^*) - 1) = 2(1 - \Phi(z^*)),$$

so that

$$\Phi(z^*) = 1 - \alpha/2 \implies z^* = z_{\alpha/2}.$$

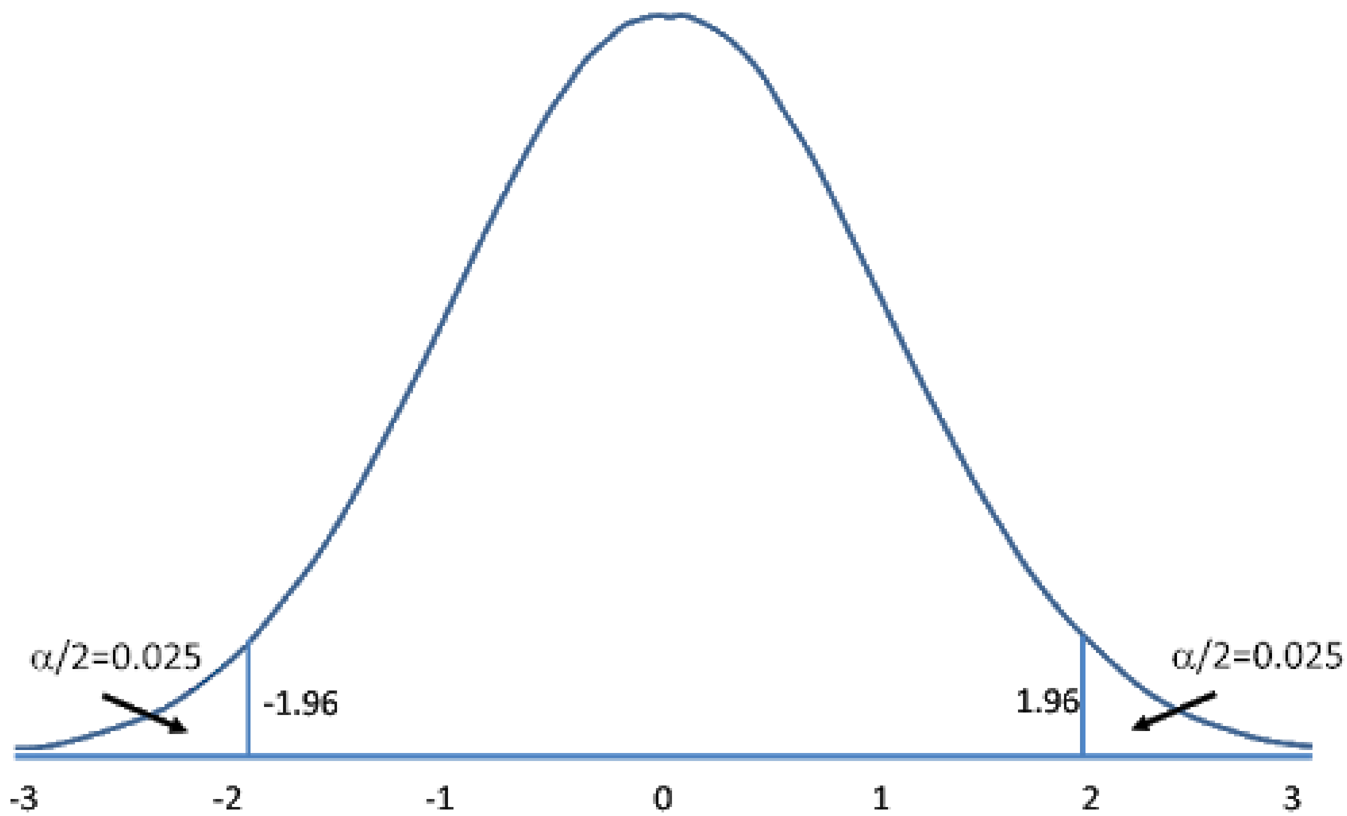

$1-\alpha$
$\alpha/2$
$\alpha/2$
$-z_{\alpha/2}$
0
$z_{\alpha/2}$
$z$

For instance,

$$P(|Z| > z_{0.025}) = 0.05 \implies z_{0.025} = 1.96$$
$$P(|Z| > z_{0.005}) = 0.01 \implies z_{0.005} = 2.575.$$

The symmetric $100(1-\alpha)\%$ confidence interval can generally be written as

$$\overline{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} < \mu < \overline{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \implies \overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$


α/2=0.025
-1.96
1.96
α/2=0.025
-3
-2
-1
0
1
2
3

For a given confidence level $\alpha$, **shorter confidence intervals are better** in relation to estimating the mean:

- estimates become better when the sample size $n$ increases;
- estimates become better when $\sigma$ decreases.

If $\alpha_1 > \alpha_2$, the $100(1-\alpha_1)\%$ C.I. is smaller than the $100(1-\alpha_2)\%$ C.I. (i.e. a $95\%$ C.I. is always shorter than a $99\%$ C.I.)

If the sample comes from a normal population, then the C.I. is **exact**. Otherwise, if $n$ is large, we may use the CLT and get an **approximate** C.I.

## Examples:

1. A sample of $9$ observations from a normal population with known standard deviation $\sigma = 5$ yields a sample mean $\overline{X} = 19.93$. Provide a $95\%$ and a $99\%$ C.I. for the unknown population mean $\mu$ based on this sample.

   **Solution:** the estimate of $\mu$ is $\overline{X} = 19.93$. The $100(1-\alpha)\%$ confidence intervals are
$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}.$$

$$95\% : \overline{X} \pm z_{0.025}\frac{\sigma}{\sqrt{n}} \Rightarrow 19.93 \pm 1.96\frac{5}{\sqrt{9}} \Rightarrow 19.93 \pm 3.27 \text{ or } (16.66, 23.20)$$
$$99\% : \overline{X} \pm z_{0.005}\frac{\sigma}{\sqrt{n}} \Rightarrow 19.93 \pm 2.575\frac{5}{\sqrt{9}} \Rightarrow 19.93 \pm 4.29 \text{ or } (15.64, 24.22)$$

2. A sample of $25$ observations from a normal population with known standard deviation $\sigma = 5$ yields a sample mean $\overline{X} = 19.93$. Provide a $95\%$ and a $99\%$ C.I. for the unknown population mean $\mu$ based on this sample.

   **Solution:** the estimate of $\mu$ is $\overline{X} = 19.93$. The $100(1-\alpha)\%$ confidence intervals are

$$95\% : \overline{X} \pm z_{0.025}\frac{\sigma}{\sqrt{n}} \Rightarrow 19.93 \pm 1.96\frac{5}{\sqrt{25}} \Rightarrow 19.93 \pm 1.96 \text{ or } (17.97, 21.89)$$

$$99\% : \overline{X} \pm z_{0.005}\frac{\sigma}{\sqrt{n}} \Rightarrow 19.93 \pm 2.575\frac{5}{\sqrt{25}} \Rightarrow 19.93 \pm 2.58 \text{ or } (17.35, 22.51)$$

3. A sample of $25$ observations from a normal population with known standard deviation $\sigma = 10$ yields a sample mean $\overline{X} = 19.93$. Provide a $95\%$ and a $99\%$ C.I. for the unknown population mean $\mu$ based on this sample.

   **Solution:** the estimate of $\mu$ is $\overline{X} = 19.93$. The $100(1-\alpha)\%$ confidence intervals are

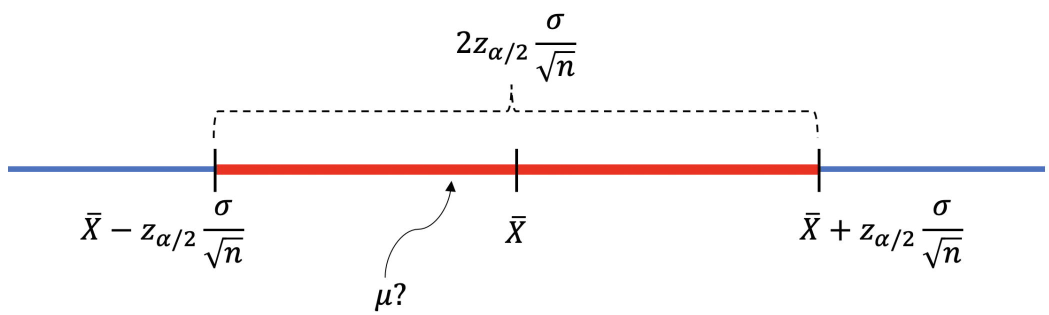
   $$95\% : \overline{X} \pm z_{0.025}\frac{\sigma}{\sqrt{n}} \Rightarrow 19.93 \pm 1.96\frac{10}{\sqrt{25}} \Rightarrow 19.93 \pm 3.92 \text{ or } (16.01, 23.85)$$

   $$99\% : \overline{X} \pm z_{0.005}\frac{\sigma}{\sqrt{n}} \Rightarrow 19.93 \pm 2.575\frac{10}{\sqrt{25}} \Rightarrow 19.93 \pm 5.15 \text{ or } (14.78, 25.08)$$

   Note how the confidence intervals are affected by $\alpha$, $n$, and $\sigma$.

# 5.3 – Choice of Sample Size

The **error** we commit by estimating $\mu$ *via* the sample mean $\overline{X}$ is smaller than $z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$, with probability $100(1-\alpha)\%$.




If we want to control the error, the only thing we can really do is control the sample size:

$$E > z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \implies n > \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2.$$

# Examples:

1. A sample $\{X_1, \ldots, X_n\}$ is selected from a normal population with standard deviation $\sigma = 100$. What sample size should be used to insure that the error on the population estimate is at most $E = 10$, at a confidence level $\alpha = 0.05$?

   **Solution:** as long as

$$n > \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{z_{0.025} \cdot 100}{10}\right)^2 = (19.6)^2 = 384.16,$$

   then the error committed by using $\overline{X}$ to estimate $\mu$ will be at most $10$, with $95\%$ probability.

2. Repeat the first example, but with $\sigma = 10$.

   **Solution:** we need

$$n > \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{z_{0.025} \cdot 10}{10}\right)^2 = (1.96)^2 = 3.8416.$$

3. Repeat the first example, but with $E = 1$.

   **Solution:** we need

$$n > \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{z_{0.025} \cdot 100}{1}\right)^2 = (196)^2 = 38416.$$

4. Repeat the first example, but with $\alpha = 0.01$.

   **Solution:** we need

$$n > \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2 = \left(\frac{z_{0.005} \cdot 100}{10}\right)^2 = (25.75)^2 = 663.0625.$$

The relationship between $\alpha$, $\sigma$, $E$, and $n$ is not always intuitive!

# 5.4 – C.I. for $\mu$ when $\sigma$ is Unknown

So far, we have been in the fortunate situation of sampling from a population with known variance $\sigma^2$.

What do we do when the population variance is **unknown**?

We estimate $\sigma$ using the **sample variance**

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

(remember that the true population mean $\mu$ is also unknown... that's what we're trying to find!) and the **sample standard deviation** $S = \sqrt{S^2}$.

If $\sigma$ is known, we know from the CLT that $\frac{\overline{X}-\mu}{\sigma/\sqrt{n}}$ is approximately $\mathcal{N}(0,1)$.

If $\sigma$ is unknown, it can be shown that $\frac{\overline{X}-\mu}{S/\sqrt{n}}$ follows approximately $t(n-1)$, the **Student $T-$distribution with $n-1$ degrees of freedom**.

Consequently, for a confidence level $\alpha$,

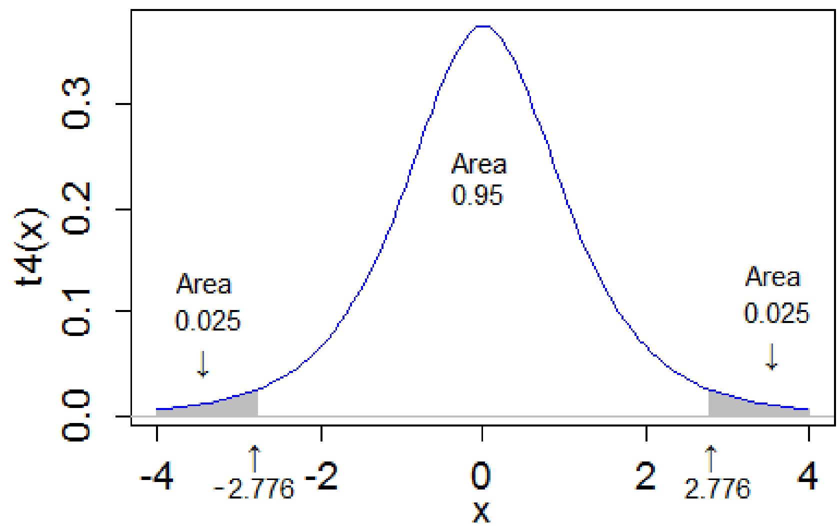
$$P\left(-t_{\alpha/2}(n-1) < \frac{\overline{X}-\mu}{S/\sqrt{n}} < t_{\alpha/2}(n-1)\right) \approx 1-\alpha,$$

where $t_{\alpha/2}(n-1)$ is the $100(1-\alpha/2)^{\text{th}}$ percentile of $t(n-1)$ (these can be read from the table). Equality is reached if the underlying population is normal.

$$100(1-\alpha)\% \text{ C.I. for } \mu : \overline{X} \pm t_{\alpha/2}(n-1)\frac{S}{\sqrt{n}}.$$

For instance, if $\alpha = 0.05$ and $\{X_1, X_2, X_3, X_4, X_5\}$ are samples from a normal distribution with unknown mean $\mu$ and unknown variance $\sigma^2$, then

$$t_{0.025}(5-1) = 2.776 \quad \text{and} \quad P\left(-2.776 < \frac{\overline{X}-\mu}{S/\sqrt{5}} < 2.776\right) = 0.95.$$



## Examples:

1. For a given year, $9$ measurements of ozone concentration are obtained:

   `3.5 5.1 6.6 6.0 4.2 4.4 5.3 5.6 4.4`

   Assume that the measured ozone concentrations follow a normal distribution with variance $\sigma^2 = 1.21$, build a $95\%$ C.I. for the population mean $\mu$. Note that $\overline{X} = 5.01$ and that $S = 0.97$.

   **Solution:** since we know the variance, we need to use the standard normal percentile $z_{\alpha/2} = z_{0.025} = 1.96$ :

$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 5.01 \pm 1.96\frac{\sqrt{1.21}}{\sqrt{9}} = 5.01 \pm 0.72 \text{ or } (4.29, 5.73).$$

2. Same thing, but assume that the variance of the underlying population is unknown.

   **Solution:** since we do not know the variance, we need to use the Student percentile $t_{\alpha/2}(n-1) = t_{0.025}(8) = 2.306$ (make sure you understand how to get this value from the table):

$$\overline{X} \pm t_{\alpha/2}(n-1)\frac{S}{\sqrt{n}} = 5.01 \pm 2.306\frac{0.97}{\sqrt{9}} \text{ or } (4.26, 5.76).$$

   The $95\%$ C.I. when we know the variance is **tighter** (smaller), which is natural as we are more confident about our results when we have more information.

# 5.5 – C.I. for a Proportion

If $X \sim \mathcal{B}(n,p)$ (number of successes in $n$ trials), then the point estimator for $p$ is $\hat{P} = \frac{X}{n}$.

Recall that $\mathrm{E}[X] = np$ and $\mathrm{Var}[X] = np(1-p)$.

We can standardize any random variable:

$$Z = \frac{X-\mu}{\sigma} = \frac{n\hat{P} - np}{\sqrt{np(1-p)}} = \frac{\hat{P} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

is approximately $\mathcal{N}(0,1)$.

Thus, for sufficiently large $n$,

$$P\left(-z_{\alpha/2} < \frac{\hat{P}-p}{\sqrt{\frac{p(1-p)}{n}}} < z_{\alpha/2}\right) \approx 1-\alpha.$$

Using the previous approach, an **approximate** $100(1-\alpha)\%$ C.I. for $p$ is:

$$\hat{P} - z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}} < p < \hat{P} + z_{\alpha/2}\sqrt{\frac{p(1-p)}{n}},$$

but this is not really useful because we don't actually know $p$! Instead:

$$\hat{P} - z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}} < p < \hat{P} + z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}}.$$

## Examples:

1. Two candidates ($A$ and $B$) are running for office. A poll is conducted: $1000$ voters are selected randomly and asked for their preference: $52\%$ support $A$, while $48\%$ support their rival, $B$. Provide a $95\%$ C.I. for the support of each candidate.

   **Solution:** we use $\alpha = 0.05$ and $\hat{P} = 0.52$. The $95\%$ C.I. for $A$ is

$$\hat{P} \pm z_{\alpha/2}\sqrt{\frac{\hat{P}(1-\hat{P})}{n}} = 0.52 \pm 1.96\sqrt{\frac{0.52 \cdot 0.48}{1000}} \approx 0.52 \pm 0.031.$$

   The $95\%$ C.I. for $B$ is $0.48 \pm 0.031$.

2. On the strength of this polling result, a newspaper prints the following headline: "Candidate $A$ Leads Candidate $B$!" Is the headline warranted?

   **Solution:** although there is a $4-$point gap in the poll numbers, the true support for candidate $A$ is in the $48.9\% - 55.1\%$ range, and, the true support for candidate $B$ is in the $44.9\% - 51.1\%$ range, with probability $95\%$ (that is to say, $19$ times out of $20$).

   Since there is overlap in the confidence intervals, the race is more likely to be a dead heat.

# Appendix – Summary

**Sample:** $\{X_1, \ldots, X_n\}$. **Objective:** predict $\mu$ with confidence level $\alpha$.

- If population is **normal** with **known** variance $\sigma^2$, the **exact** $100(1-\alpha)\%$ C.I. is
$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}.$$

- If population is **non-normal** with **known** variance $\sigma^2$ and $n$ is '**big**', the **approximate** $100(1-\alpha)\%$ C.I. is
$$\overline{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}.$$

- If population is **normal** with **unknown** variance, the **exact** $100(1-\alpha)\%$ C.I. is
$$\overline{X} \pm t_{\alpha/2}(n-1)\frac{S}{\sqrt{n}}.$$

- If population has **unknown** variance and $n$ is '**big**', the **approximate** $100(1-\alpha)\%$ C.I. is
$$\overline{X} \pm z_{\alpha/2}\frac{S}{\sqrt{n}}.$$

- If population has **unknown** variance and $n$ is '**small**', you are S.O.O.L.