

MAT 2377 – Assignment 2

Deadline: [Tuesday Mar 16, 2021 at 3:00 pm](#)

Total = 100 marks

Please complete **all** the questions in this assignment. However, due to our limited TA resources, it is possible that not all the questions will be marked. You will not be informed beforehand which questions will be marked.

Please follow the instructions on course syllabus to submit your assignment online. **Late assignments will not be accepted.**

1. About 1% of a certain type of light bulb fails during a 24-hour test. Failures are assumed to be independent. Consider a sign consisting of 10 such light bulbs. Let X be the number of light bulbs that fail during the 24-hour test.

- (a) What distribution does X follow?
- (b) What is the probability that the sign will burn for the full 24-hour test with no bulb failure?
- (c) What is the probability that the sign will lose at least 3 bulbs during the 24-hour test?
- (d) What is the probability that the sign will lose at least 2 and at most 4 bulbs during the test?

2. Water samples in European rivers are selected one at a time. Each sample has an independent probability of 3% of testing positive for algae blooms.

- (a) What is the probability that the 7th sample is the first to test positive for algae blooms?
- (b) Let T_2 be the number of samples required to obtain 2 positive algae blooms tests. What distribution does T_2 follow? What is the expected value and the standard deviation of T_2 ?
- (c) What is the probability that the 5th sample is the second to test positive for algae blooms?

3. Suppose that a certain type of magnetic tape contains, on the average, 2 defects per 100 meters, according to a Poisson process.

- (a) What is the probability that the next 100 meters of tape contain x defects, where $x = 0, 1, 2, \dots$?
- (b) What is the expected value and standard deviation of the number of defects per 100m?
- (c) What is the expected number of defects in the next 300 meters of tape?
- (d) What is the probability that there are more than 2 defects between meters 20 and 75?

4. The lifetime (in years) of a specific helicopter part follows an exponential distribution whose expectation is 3.2 years.

- (a) What is the probability that one of these parts will stay operational for more than 4.4 years?
- (b) What is the probability that one of these parts will stay operational between 3 and 9 months?
- (c) If one of these parts is still operational after 3 years, what is the probability that it will remain operational for another 2 years?

5. Suppose that the number of cars passing a certain point of some road per minute between 8am and 10am on a Sunday morning follows a Poisson distribution with mean $\lambda = 5$. A traffic inspector has arrived at the specified location during the aforementioned period. She notes the wait time W_1 for the next car to pass by the location.

- (a) What distribution does W_1 follow? What are its expectation and variance?
- (b) What is the probability she needs to wait more than 1 minute before the next car passes by?
- (c) What is the probability that she needs to wait more than 1 minute for 2 cars to pass by?

6. Iron plates are required to have a certain thickness but each plate produced will differ slightly from each other due to properties of the material and uncertainties in the behaviour of the machines that make them. Let X be the plate thickness in mm of plates produced by a given machine. Using the machine's default setting, X follows a normal distribution with mean 10mm and standard deviation 0.02mm.

- (a) What percentage of plates should be expected to be thinner than 0.97mm?
- (b) What percentage of plates should be expected to be thicker than 10.05mm?
- (c) What percentage of plates should be expected to deviate in thickness by more than 0.03mm from 10.00mm?
- (d) Find $c > 0$ to ensure that 5% of plates are expected to deviate in thickness by more than c mm from 10.00mm?
- (e) Given the c found in part (d), what percentage of plates are expected to deviate by more than c mm from 10.00mm if a slight adjustment in the machine shifts the expected value of X to 10.01mm.