MAT 2377
Probability and Statistics for Engineers

Practice Set

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Winter 2021

Based on course notes by Rafał Kulik

Q72. In a box of 50 fuses there are 8 defective ones. We choose 5 fuses randomly (without replacement). What is the probability that all 5 fuses are not defective?

a) 0.4015 b) 0.84 c) 0.3725 d) 0.4275 e) none of the preceding

Solution: If we assume that the defective fuses are independently found in the box, then the probability of a fuse being defective is $p = 8/50 = 0.16$.

Assume we sample $n = 5$ fuses randomly. Let X be the number of defective fuses in the sample. Then $X \sim \mathcal{B}(5, 0.16)$.

We are looking for

$$P(X = 0) = \binom{5}{0}(0.16)^0(1 - 0.16)^{5-0} = 0.84^5 \approx 0.4182.$$

Q35. Samples of 20 parts from a metal punching process are selected every hour. Typically, 1% of the parts require re-work. Let X denote the number of parts in the sample that require re-work. A process problem is suspected if X exceeds its mean by more than three standard deviations.

a) What is the probability that there is a process problem?

b) If the re-work percentage increases to 4%, what is the probability that X exceeds 1?

c) If the re-work percentage increases to 4%, what is the probability that X exceeds 1 in at least one of the next five sampling hours?

Solution:

a) We have $X \sim \mathcal{B}(n, p)$, $n = 20$, $p = 0.01$ (where a trial success = a part requires re-work). By definition of the binomial distribution, $\mathrm{E}[X] = np = 0.2$, $\mathrm{Var}[X] = np(1-p) = 0.2 \times 0.99 = 0.198$, and $\mathrm{SD}[X] = \sqrt{\mathrm{Var}[X]} \approx 0.44$.

What we want to compute is $P(X > \mathrm{E}[X] + 3 \cdot \mathrm{SD}[X])$. Thus,

$$
\begin{aligned}
P(X > 0.2 + 3 \cdot 0.44) &= P(X > 1.535) = P(X \geq 2) \\
&= 1 - P(X \leq 1) = 1 - (P(X = 0) + P(X = 1)) \\
&= 1 - \binom{20}{0} 0.01^0 \cdot 0.99^{20} - \binom{20}{1} 0.01^1 \cdot 0.99^{19} \\
&(= 1 - \texttt{pbinom}(1, 20, 0.01)) \approx 0.017.
\end{aligned}
$$

Alternatively, you may work with $Y \sim \mathcal{B}(n, 0.99)$, where Y is the # of samples which do not require re-work.

b) We have the same set-up, but with $p = 0.04$. We are still interested in

$$P(X > 1) = P(X \geq 2) = 1 - P(X \leq 1) = 1 - (P(X = 0) + P(X = 1))$$
$$= 1 - \binom{20}{0} 0.04^0 \cdot 0.96^{20} - \binom{20}{1} 0.04^1 \cdot 0.96^{19}$$
$$(= \texttt{1 - pbinom(1, 20, 0.04)}) \approx 0.19.$$

c) Now, we have 5 hourly samples, and each of them consists of 20 items. let W be the number of hourly samples where the number of items which require re-work is larger than 1 (where a trial success = hourly sample has more than 1 defective item, i.e. $X > 1$).

We have $W \sim \mathcal{B}(5, p_0)$, where $p_0 = P(X > 1) = 0.19$ is the probability of a trial success. Thus, we need to evaluate

$$P(W \geq 1) = 1 - P(W = 0) = 1 - \binom{5}{0} 0.19^0 \cdot 0.81^5 \approx 0.651.$$

This example highlights the procedure for problems of this nature: we identify the appropriate distribution model; we evaluate its parameters; we identify the appropriate probability to evaluate, and we compute its value.