# MAT 2377
# Probability and Statistics for Engineers

## Practice Set

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Based on course notes by Rafał Kulik

**Q91**. Use the data from **Q89**.

a) Provide a the $5-$number summary of the data $(q_0, q_1, q_2, q_3, q_4)$, as well as the interquartile range IQR.

b) Display the $5-$number summary as a boxplot chart.

## Solution:

a) To make it easy to compute the quartiles, we provide an **ordered stem-and-leaf diagram** of the concentrations on the next slide. There were 100 observations, so

$$Q_0 = 0.85,\ Q_1 = 0.89,\ Q_2 = 0.92,\ Q_3 = 0.97,\ Q_4 = 1.06.$$

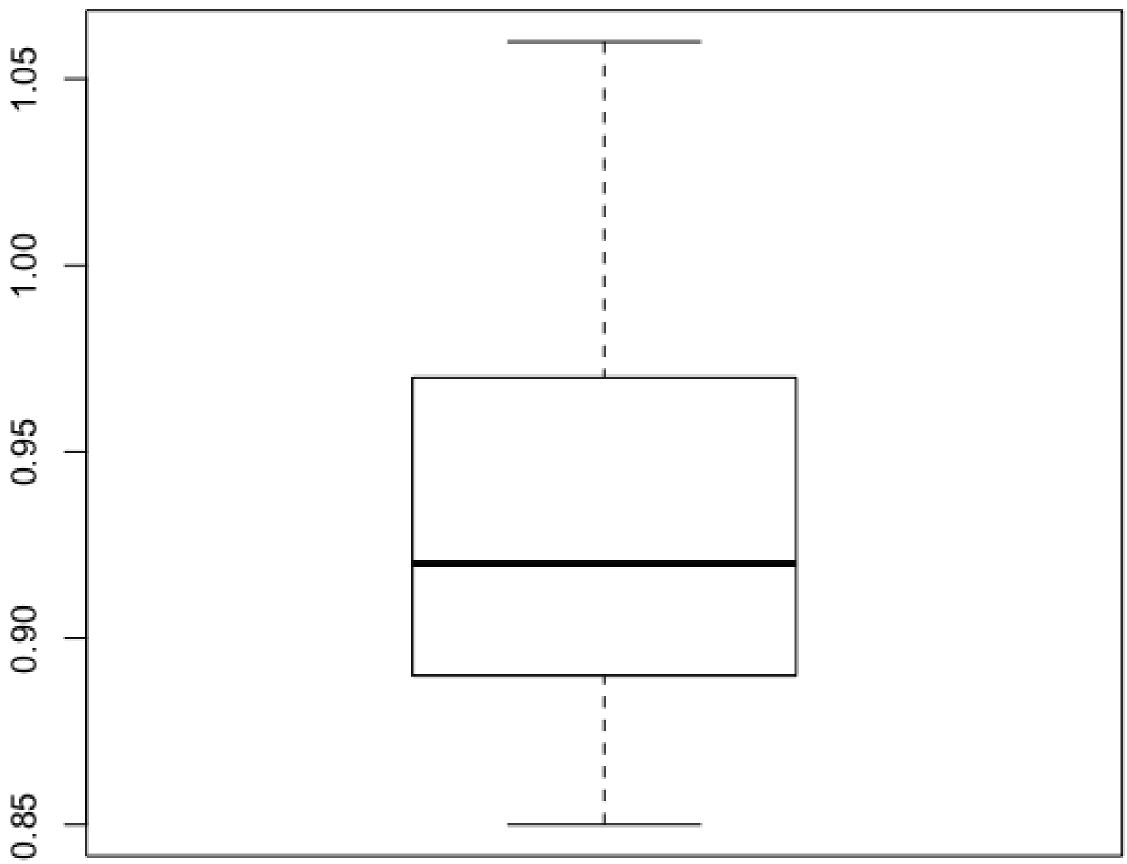
The IQR, meanwhile, is $Q_3 - Q_1 = 0.08$.

b) There are no outliers since

$$Q_1 - 1.5 \times \text{IQR} = 0.85 - 1.5(0.08) < Q_0,\ \text{and}$$
$$Q_3 + 1.5 \times \text{IQR} = 0.97 + 1.5(0.08) > Q_4.$$

**Table 6.2-6** Ordered stem-and-leaf diagram of fluoride concentrations

| Stems | Leaves | Frequency |
|---|---|---|
| **0.8***f* | 5 5 5 5 | 4 |
| **0.8***s* | 6 6 6 7 7 7 7 | 7 |
| **0.8•** | 8 8 8 8 8 8 8 8 8 9 9 9 9 9 9 9 | 16 |
| **0.9**∗ | 0 0 0 0 0 0 0 0 0 1 1 1 1 1 1 1 1 | 17 |
| **0.9***t* | 2 2 2 2 2 2 2 2 2 2 3 3 3 3 3 3 3 3 3 | 19 |
| **0.9***f* | 4 4 5 5 5 5 5 5 5 | 9 |
| **0.9***s* | 6 6 7 7 7 7 | 6 |
| **0.9•** | 8 8 8 8 8 8 8 8 9 9 | 10 |
| **1.0**∗ | 0 0 0 0 1 1 1 | 7 |
| **1.0***t* | 2 3 3 | 3 |
| **1.0***f* | 5 | 1 |
| **1.0***s* | 6 | 1 |


1.05
1.00
0.95
0.90
0.85

**Q92**. Use the data from **Q89**. Compute the **midrange** $\frac{1}{2}(Q_0 + Q_4)$, the **trimean** $\frac{1}{4}(Q_1 + 2Q_2 + Q_3)$, and the **range** $Q_4 - Q_0$ for the fluoride data.

**Solution:** The midrange is $\frac{1}{2}(Q_0 + Q_4) = \frac{1}{2}(0.85 + 1.06) = 0.955.$

The trimean is $\frac{1}{4}(Q_1 + 2Q_2 + Q_3) = \frac{1}{4}(0.89 + 2(0.92) + 0.97) = 0.925.$

The range is $Q_4 - Q_0 = 1.06 - 0.85 = 0.21.$

**Q125**. Consider the following dataset:

```
2.6  3.7  0.8  9.6  5.8 -0.8  0.7 0.6
4.8  1.2  3.3  5.0  3.7  0.1 -3.1 0.3
```

The median and the interquartile range of the sample are, respectively:

a) $2.4$, $3.3$ b) $1.9$, $3.8$ c) $1.9$, $1.8$ d) $2.9$, $12.2$ e) none of the preceding

**Solution:** the correct answer is a).

**Q128**. Assume that random variables $\{X_1, \ldots, X_8\}$ follow a normal distribution with mean $2$ and variance $24$. Independently, assume that random variables $\{Y_1, \ldots, X_{16}\}$ follow a normal distribution with mean $1$ and variance $16$. Let $\overline{X}$ and $\overline{Y}$ be the corresponding sample means. Then $P(\overline{X} + \overline{Y} > 4)$ is:

a)$0.7721$ b)$0.30855$ c)$0.69165$ d)$0.9883$ e)none of the preceding

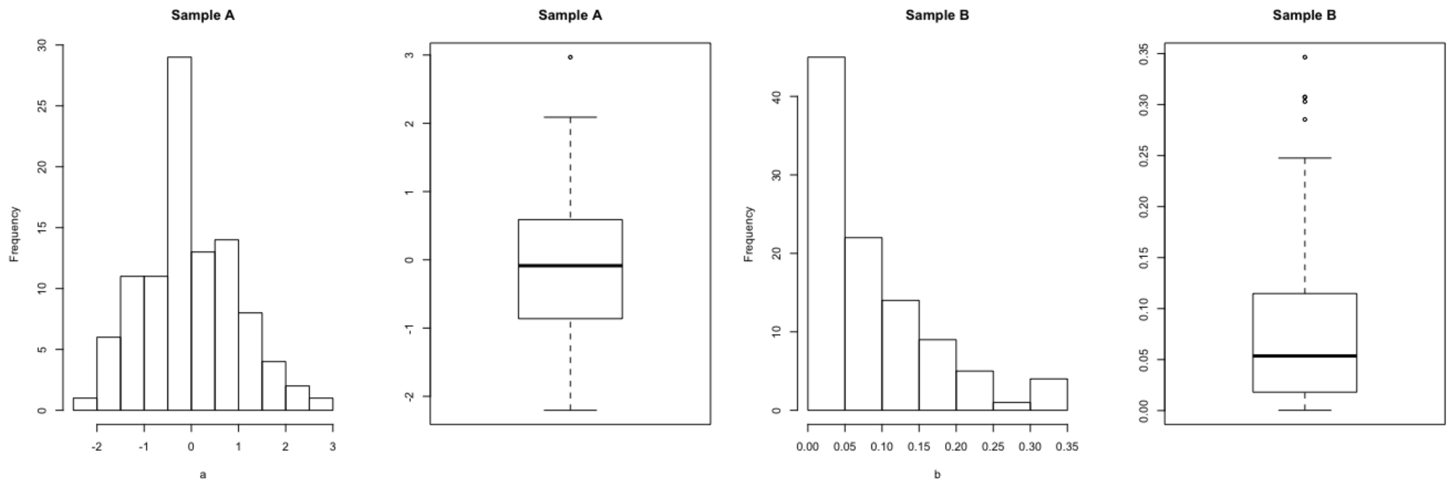
**Solution:** since the $X_i$ and $Y_j$ are independent,

$$\overline{X} + \overline{Y} \sim \mathcal{N}\left(2+1, \frac{24}{8} + \frac{16}{16}\right) = \mathcal{N}(3, 4).$$

Thus

$$P(\overline{X} + \overline{Y} > 4) = P\left(\frac{\overline{X} + \overline{Y} - 3}{\sqrt{4}} > \frac{4-3}{\sqrt{4}}\right) = P(Z > 0.5)$$
$$= 1 - P(Z < 0.5) = 1 - \Phi(0.5) \approx 1 - 0.6915 = 0.3085.$$

**Q136**. The following charts show a histogram and a boxplot for two samples, $A$ and $B$. Based on these charts, we may conclude that




a)only $A$ arises from a normal population

b)only $B$ arises from a normal population

c)both $A$ and $B$ arise from a normal population

**Solution:** it is reasonable to expect that $A$ arises from a normal population, but the skew and asymmetric distribution for $B$ means it does not come from a normal distribution.