MAT 2377
Probability and Statistics for Engineers

Practice Set

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105. A company is currently using titanium alloy rods it purchases from supplier A. A new supplier (supplier B) approaches the company and offers the same quality (at least according to supplier B's claim) rods at a lower price. The company is certainly interested in the offer. At the same time, the company wants to make sure that the safety of their product is not compromised. The company randomly selects ten rods from each of the lots shipped by suppliers A and B and measures the yield strengths of the selected rods. The observed sample mean and sample standard deviation are 651 MPa and 2 MPa for supplier's A rods, respectively, and the same parameters are 657 MPa and 3 MPa for supplier B's rods. Perform the test $H_0 : \mu_A = \mu_B$ against $\mu_A \neq \mu_B$. Use $\alpha = 0.05$. Assume that the variances are equal but unknown.

Solution: This is a two-sample test: $H_0 : \mu_A = \mu_B$, $H_1 : \mu_A \neq \mu_B$. We have $\overline{x}_1 = 651$, $\overline{x}_2 = 657$, $s_1 = 2$, $s_2 = 3$. The observed difference in means is $\overline{x}_1 - \overline{x}_2 = -6$. The test statistic is

$$T_0 = \frac{\overline{x}_1 - \overline{x}_2}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{n_1+n_2-2},$$

where S_p^2 is the **pooled variance** which is computed as follows:

$$S_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = 6.5.$$

We compute the p-value as

$$2P(\overline{x}_1 - \overline{x}_2 \leq -6) = 2P\left(\frac{\overline{x}_1 - \overline{x}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \leq -5.26\right) = 2P(t_{18} < -5.26)$$
$$= 2P(t_{18} > 5.26) < 2(0.0005) = 0.001.$$

This is smaller than $\alpha = 0.05$, thus we reject H_0 in favour of H_1 at level $\alpha = 0.05$.

106. The deflection temperature under load for two different types of plastic pipe is being investigated. Two random samples of 15 pipe specimens are tested, and the deflection temperatures observed are as follows:

Type 1: $206, 188, 205, 187, 194, 193, 207, 185, 189, 213, 192, 210, 194, 178, 205.$

Type 2: $177, 197, 206, 201, 180, 176, 185, 200, 197, 192, 198, 188, 189, 203, 192.$

Does the data support the claim that the deflection temperature under load for type 1 pipes exceeds that of type 2? Calculate the p-value, using $\alpha = 0.05$, and state your conclusion.

Solution: for this $2-$sample test, we test $H_0 : \mu_1 = \mu_2$ vs. $H_0 : \mu_1 > \mu_2$. We have $\overline{x}_1 = 196.4$, $\overline{x}_2 = 192.0667$, $s_1^2 = 109.8286$, $s_2^2 = 89.06667$, $n_1 = n_2 = 15$, and

$$s_p^2 = \frac{(15-1)109.8286 + (15-1)89.06667}{15+15-2} = 99.44762.$$

We are in Case 2 (σ_1^2, σ_2^2 unknown, small samples), so the test statistic is

$$T_0 = \frac{\overline{X}_1 - \overline{X}_2}{S_p\sqrt{1/n_1 + 1/n_2}} \sim t(n_1 + n_2 - 2).$$

The observed value of the test statistic is

$$t_0 = \frac{\overline{x}_1 - \overline{x}_2}{s_p\sqrt{1/n_1 + 1/n_2}} = 1.19.$$

From the $t-$table we get $t_{0.05}(28) = 1.701$, so that $t_0 < t_{0.05}(28)$, meaning that we cannot reject H_0; there is no evidence that the deflection temperature under load for type 1 pipes exceeds that for type 2 pipes. The p-value is

$$P(t(28) > 1.19) \in (0.1, 0.25),$$

since $p(t(28) > 1.313) = 0.1$ and $P(t(28) > 0.683) = 0.25$.

Q107. It is claimed that 15% of a certain population is left-handed, but a researcher doubts this claim. They decide to randomly sample 200 people and use the anticipated small number to provide evidence against the claim of 15%. Suppose 22 of the 200 are left-handed. Compute the $p-$value associated with the hypothesis (assuming a binomial distribution), and provide an interpretation.

Solution: we assume that the binomial distribution is appropriate. Let X denote the (random, i.e. before observing) number of left-handed people in the sample, and let p denote the true proportion of left-handed people in the population. We can set up the formal hypothesis test as follows:

- Model: $X \sim \mathcal{B}(200, p)$, where p is the true proportion.
- $H_0 : p = 0.15$ (claim), against $H_1 : p < 0.15$ (suspicion: $12/200 = 11\%$)
- Evidence against H_0: small values of X. Observed value: 22.
- $p-$value: $P(X \leq 22)$ under $X \sim \mathcal{B}(200, 0.15)$ (i.e. when H_0 true). But $P(X \leq 22) = \texttt{pbinom}(22, 200, 0.15) \approx 0.0645$. The “small-ish“ $p-$value provides some evidence against the claim of 15%.

Q108. A child psychologist believes that nursery school attendance improves children's social perceptiveness (SP). They use 8 pairs of twins, randomly choosing one to attend nursery school and the other to stay at home, and then obtains scores for all 16. In 6 of the 8 pairs, the twin attending nursery school scored better on the SP test. Compute the $p-$value associated with the hypothesis (assuming a binomial distribution), and provide an interpretation.

Solution:

- Model $X \sim \mathcal{B}(8, p)$, where X is $\#$ of pairs in the sample where the twin attending nursery school scored better, and p is the true probability that a twin attending nursery school scores better

- H_0: "Attending nursery school has no effect on SP", $H_0 : p = 0.5$, against H_1: "Attending nursery school improves SP". $H_1 : p > 0.5$

- If H_0 is true, $X \sim \mathcal{B}(8, 0.5)$; if H_1 true, X would tend to take **larger values** than it would under H_0. Thus larger values of X provide more evidence against H_0 (in the direction of H_1).

- The $p-$value under H_0 is $P\left(X \geq 6\right), \quad X \sim \mathcal{B}(8, 0.5)$. The $p-$value is

$$P\left(X \geq 6\right) = 1 - P(X \leq 5) = \texttt{pbinom}(6, 8, 0.5) = 0.1445.$$

Interpretation: if there was no real effect, we would see 6 or more improvements out of 8 around 14% of the time, just by chance (which is fairly large, all things considered). The data does not provide compelling evidence against H_0, the null hypothesis (no effect). Consequently, the researcher cannot convince us that attending nursery school improves SP.