MAT 2377
Probability and Statistics for Engineers

Practice Set

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Based on course notes by Rafał Kulik

Q118. The thickness of a plastic film (in mm) on a substrate material is thought to be influenced by the temperature at which the coating is applied. A completely randomized experiment is carried out. 11 substrates are coated at 125F, resulting in a sample mean coating thickness of $\overline{x}_1 = 103.5$ and a sample standard deviation of $s_1 = 10.2$. Another 11 substrates are coated at 150F, for which $\overline{x}_2 = 99.7$ and $s_2 = 11.7$ are observed. We want to test equality of means against the two-sided alternative. The value of the appropriate test statistics and the decision are (for $\alpha = 0.05$):

a)0.81; Reject H_0.

b)0.81; Do not reject H_0.

c)1.81; Reject H_0.

d)1.81; Do not reject H_0.

e)none of the preceding

Note: assume that population variances are unknown but equal.

Solution: this is two-sample test with small samples and unknown variances, so we need the pooled variance

$$s_p^2 = \frac{(11-1)10.2^2 + (11-1)11.7^2}{11+11-2} = 120.465,$$

or $s_p = 10.97$. The observed value of the test statistic is

$$t_0 = \frac{\overline{x}_1 - \bar{x}_2}{s_p\sqrt{1/n_1 + 1/n_2}} = \frac{103.5 - 99.7}{10.97\sqrt{1/101 + 1/11}} = 0.81.$$

Since $t_{0.05/2}(11+11-2) = 2.086 > t_0$, we don't reject H_0.

Q121. A company claims that the mean deflection of a piece of steel which is 10ft long is equal to 0.012ft. A buyer suspects that it is bigger than 0.012ft. The following data x_i has been collected:

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0.0132 0.0138 0.0108 0.0126 0.0136 0.0112 0.0124 0.0116 0.0127 0.0131
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Assuming normality and that $\sum_{i=1}^{10} x_i^2 = 0.0016$, what are the $p-$value for the appropriate one-sided test and the corresponding decision?

a)$p \in (0.05, 0.1)$ and reject H_0 at $\alpha = 0.05$.

b)$p \in (0.05, 0.1)$ and do not reject H_0 at $\alpha = 0.05$.

c)$p \in (0.1, 0.25)$ and reject H_0 at $\alpha = 0.05$.

d)$p \in (0.1, 0.25)$ and do not reject H_0 at $\alpha = 0.05$.

e)none of the preceding

Solution: we test for $H_0 : \mu = 0.012$ against $H_1 : \mu > 0.012$. As the variance of the underlying population is unknown, we will be using the one-sided $t-$test. The estimated sample variance is

$$S^2 = \frac{1}{10-1}\left(\sum_{i=1}^{10} x_i^2 - \frac{1}{10}\left(\sum_{i=1}^{10} x_i\right)^2\right) = 0.00000102.$$

The observed mean is $\overline{x} = 0.0125$. We calculate the corresponding $p-$value:

$$P(\overline{X} > \overline{x}) = P\left(\frac{\overline{X}-\mu_0}{S/\sqrt{n}} > \frac{\overline{x}-\mu_0}{S/\sqrt{n}}\right) = P\left(\frac{\overline{X}-\mu_0}{S/\sqrt{n}} > \frac{0.0125-0.012}{\sqrt{0.00000102/10}}\right)$$

$$= P(t(10-1) > 1.5638) = P(t(9) > 1.5638) \in (0.05, 0.1)$$

and we do not reject H_0 at $\alpha = 0.05$.

Q122. In an effort to compare the durability of two different types of sandpaper, 10 pieces of type A sandpaper were subjected to treatment by a machine which measures abrasive wear; 11 pieces of type B sandpaper were subjected to the same treatment. We have the following observations:

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xA 27 26 24 29 30 26 27 23 28 27
xB 24 23 22 27 24 21 24 25 24 23 20
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Note that $\sum x_{A,i} = 267$, $\sum x_{B,i} = 257$, $\sum x_{A,i}^2 = 7169$, $\sum x_{B,i}^2 = 6041$. Assuming normality and equality of variances in abrasive wear for A and B, we want to test for equality of mean abrasive wear for A and B. The appropriate $p-$value is

a) $p < 0.01$ b) $p > 0.2$ c) $p \in (0.01, 0.05)$

d) $p \in (0.1, 0.2)$ e) $p \in (0.05, 0.1)$ f) none of the preceding

Solution: this is a two sample test. We test for $H_0 : \mu_A = \mu_B$ against $H_1 : \mu_A \neq \mu_B$. We compute $s_A^2 = 4.45$, $s_B^2 = 3.65$, $s_p^2 = 4.03$, $\overline{x}_A = 26.71$, $\overline{x}_B = 23.26$. The $p-$value is

$$
\begin{aligned}
2P(\overline{X}_A - \overline{X}_B > \overline{x}_A - \overline{x}_B) &= 2P\left(\frac{\overline{X}_A - \overline{X}_B}{S_p\sqrt{1/n_A + 1/n_B}} > \frac{3.34}{\sqrt{4.03}\sqrt{1/10+1/11}}\right) \\
&= 2P(t(10+11-2) > 3.8037) \\
&= 2P(t(19) > 3.8037) < 0.01,
\end{aligned}
$$

because $P(t(19) > 3.8037) < 0.005$.

Q129. A medical team wants to test whether a particular drug decreases diastolic blood pressure. Nine people have been tested. The team measured blood pressure before (X) and after (Y) applying the drug. The corresponding means were $\overline{X} = 91$, $\overline{Y} = 87$. The sample variance of the differences was $S_D^2 = 25$. The $p-$value for the appropriate one-sided test is between:

a)0 and 0.025

b)0.025 and 0.05

c)0.05 and 0.1

d)0.1 and 0.25

e)0.25 and 1

f) none of the preceding

Solution: this is a one-sided paired $t-$test, $H_0 : \mu_X = \mu_Y$ against $H_1 : \mu_X > \mu_Y$. The observed difference of the means is $\overline{d} = 4$. The associated $p-$value is is

$$P(\overline{D} \geq \overline{d}) = P(\overline{D} \geq 4) = P\left(\frac{\overline{D}}{S_D/\sqrt{n}} \geq \frac{4}{5/3}\right)$$
$$= P(t(n-1) > 2.4) = P(t(8) > 2.4) < 0.025,$$

since $P(t(8) > 2.4) \in (0.01, 0.025)$ according to the table.