MAT 2377
Probability and Statistics for Engineers

Practice Set

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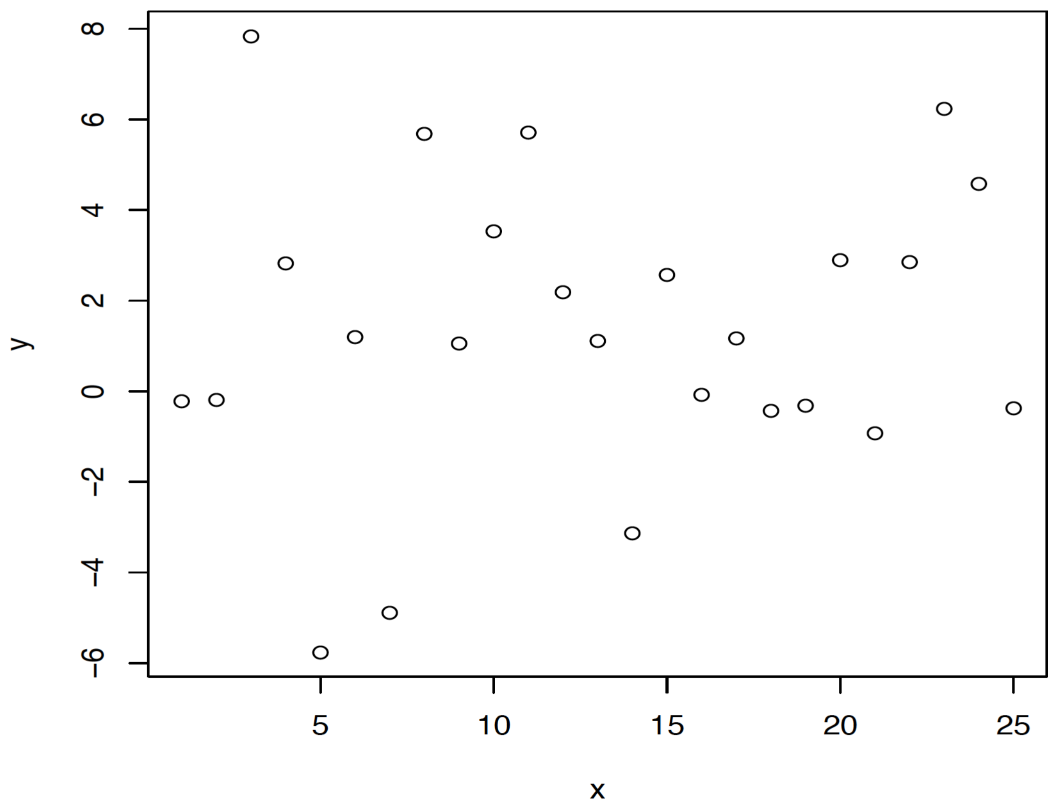
Based on course notes by Rafał Kulik

Q141. For the following data the correlation coefficient is most likely to be

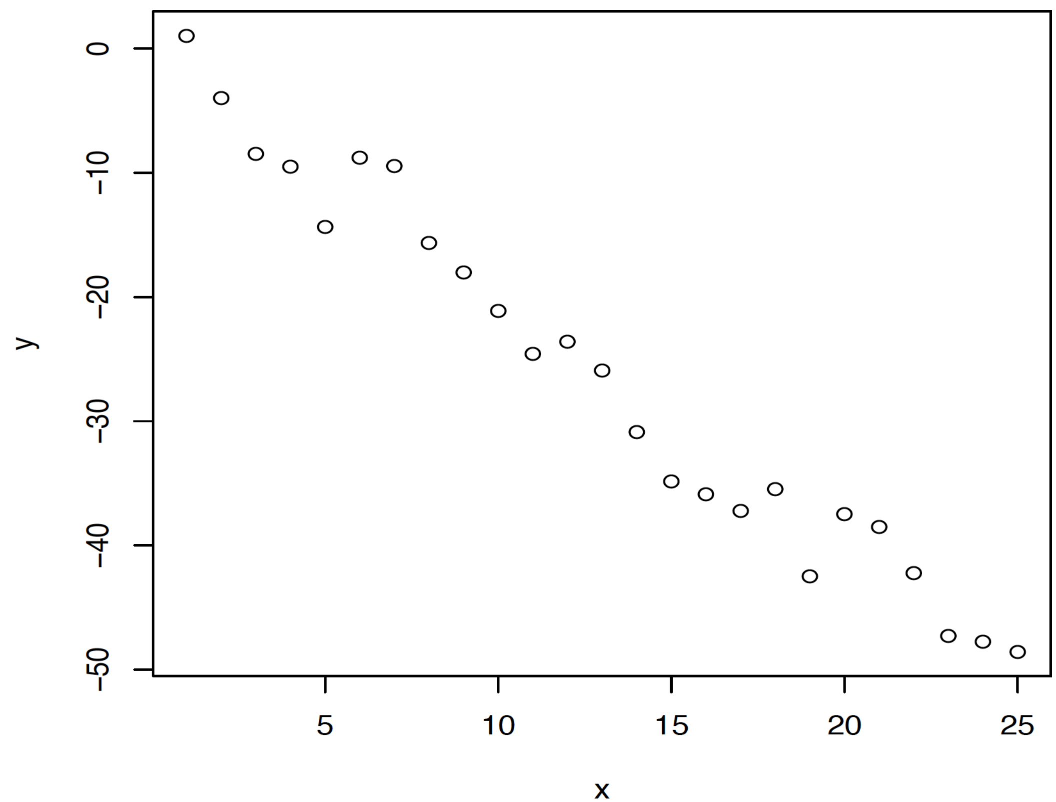
a) 0.01 b) 0.98 c) -0.5 d) -0.98



Solution: the scatterplot shows no real structure or relationship between x and y. The most likely answer si $\rho = 0.01$.

Q142. For the following data the correlation coefficient is most likely to be

a)0.01 b)0.98 c)-0.5 d)-0.98



Solution: the scatter plot shows a clear anti-correlated pattern between x and y – when x increases, y decreases and vice-versa. The most likely value is $\rho = -0.98$.

Q143. A company employs 10 part-time drivers for its fleet of trucks. Its manager wants to find a relationship between number of km driven (X) and number of working days (Y) in a typical week. The drivers are hired to drive half-day shifts, so that 3.5 stands for 7 half-day shifts.

The manager wants to use the linear regression model $Y = \beta_0 + \beta_1 x + \epsilon$ on the following data:

	1	2	3	4	5	6	7	8	9	10
x	825	215	1070	550	480	920	1350	325	670	1215
y	3.5	1.0	4.0	2.0	1.0	3.0	4.5	1.5	3.0	5.0

Note that $\sum x_i^2 = 7104300$, $\sum y_i^2 = 99.75$, and $\sum x_i y_i = 26370$. What is the fitted regression line?

Solution: we have

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 = 1297860$$

and

$$S_{xy} = 4653,$$

so that

$$b_1 = S_{xy}/S_{xx} = 0.0036,$$

and

$$b_0 = \sum_{i=1}^{n} y_i/n - b_1 \sum_{i=1}^{n} x_i = 0.1181;$$

hence the fitted line is $\hat{y} = 0.1181 + 0.0036x$.

Q144. Using the data from question **Q143**, what value is the correlation coefficient of x and y closest to?

a) 0.437 b) 0.949 c) 0.113 d) 1.123 e) none of the preceding

Solution: as in question **Q143**, we have $S_{xx} = 12978600$ and $S_{xy} = 4653$. Furthermore, we have

$$S_{yy} = \sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2 = 18.525,$$

so that the correlation coefficient is

$$\rho_{xy} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}} = \frac{4653}{\sqrt{18.525 \cdot 1297860}} \approx 0.949$$

Q145. We want to test significance of regression, i.e. $H_0 : \beta_1 = 0$ against $H_1 : \beta_1 \neq 0$. The value of the appropriate statistic and the decision for $\alpha = 0.05$ is:

a) 8.55; do not reject H_0

b) 2.31; reject H_0

c) 8.55; reject H_0

d) 2.31; do not reject H_0

e) none of the preceding

Solution: the estimated variance is

$$\hat{\sigma}^2 = \frac{S_{yy} - b_1 S_{xy}}{n-2} = \frac{1.8434}{8} = 0.23.$$

Consequently, the test statistic is

$$t_0 = \frac{b_1}{\sqrt{\hat{\sigma}^2 / S_{xx}}} = \frac{0.0036}{\sqrt{0.23/1297860}} = 8.551701.$$

Since $t_{0.05/2}(n-2) = t_{0.025}(8) = 2.306$, we reject H_0.

Q146. Regression methods were used to analyze the data from a study investigating the relationship between roadway surface temperature in F (x) and pavement defection (y). Summary quantities were $n = 20$,

$$\sum y_i = 12.75, \sum y_i^2 = 8.86, \sum x_i = 1478 \sum x_i^2 = 143,215.8 \sum x_i y_i = 1083.67.$$

a) Calculate the least squares estimates of the slope and intercept. Estimate σ^2.

b) Use the equation of the fitted line to predict what pavement deflection would be observed when the surface temperature is 90F.

c) Give a point estimate of the mean pavement deflection when the surface is 85F.

d) What change in mean pavement deflection would be expected for a 1F change in surface temperature?

Solution:

a) We have

$$b_1 = \frac{S_{xy}}{S_{xx}}, \quad b_0 = \overline{y} - b_1\overline{x}, \quad \hat{\sigma}^2 = \frac{S_{yy} - b_1 S_{xy}}{n-2},$$

where

$$\begin{aligned} S_{xy} &= \sum x_i y_i - \tfrac{1}{n}(\sum x_i)(\sum y_i) = 141.445 \\ S_{xx} &= \sum x_i^2 - \tfrac{1}{n}(\sum x_i)^2 = 33991.6 \\ S_{yy} &= \sum y_i^2 - \tfrac{1}{n}(\sum y_i)^2 = 0.731875, \end{aligned}$$

so that $b_1 = 0.00416$, $b_0 = 0.32999$, and $\hat{\sigma}^2 = 0.00797$

b) $\hat{y}(90) = b_0 + b_1 \cdot 90 = 0.70$

c) The question can be rephrased as "use the equation of the fitted line to predict what pavement deflection would be observed when the surface temperature is 85F", i.e. $\hat{y}(85) = b_0 + b_1 \cdot 85 = 0.68$.

d) That is simply the slope: $b_1 = 0.00416$

Q147. Consider the data from **Q146**.

a) Test for significance of regression using $\alpha = 0.05$. Find the p-value for this test. What conclusion can you draw?

b) Estimate the standard errors of the slope and intercept.

Solution:

a) We test for $H_0 : \beta_1 = 0$, against $H_1 : \beta_1 \neq 0$. The test statistic is $T_0 = \frac{b_1 - 0}{\sqrt{\hat{\sigma}^2\left[\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right]}} \sim t(n-2)$. Its observed value is $t_0 = \frac{b_1 - 0}{\sqrt{\hat{\sigma}^2/S_{xx}}} = 8.6$. The p-value (using $t(18)$ table) is $2P(t_{18} > 8.6) < 0.001$, and so we reject H_0 in favour of a linear relationship between x and y.

b) The standard errors are

$$\mathrm{se}(b_1) = \sqrt{\frac{\hat{\sigma}^2}{S_{xx}}}, \quad \mathrm{se}(b_0) = \sqrt{\hat{\sigma}^2\left[\frac{1}{n}+\frac{\bar{x}^2}{S_{xx}}\right]}.$$

So, $\mathrm{se}(b_1) = 0.00048$, $\mathrm{se}(b_0) = 0.04098$.